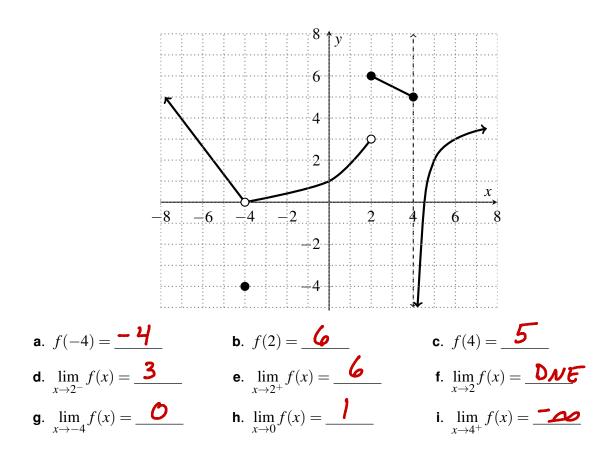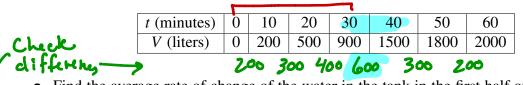Name: **Solutions**                                                    _____ / 25

**Please circle your instructor's name:**          Leah Berman          Jill Faudree

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[9 points]** Use the graph of the function of $f(x)$ to answer the following questions.



a. $f(-4) =$ **$-4$**

b. $f(2) =$ **$6$**

c. $f(4) =$ **$5$**

d. $\lim\limits_{x \to 2^-} f(x) =$ **$3$**

e. $\lim\limits_{x \to 2^+} f(x) =$ **$6$**

f. $\lim\limits_{x \to 2} f(x) =$ **DNE**

g. $\lim\limits_{x \to -4} f(x) =$ **$0$**

h. $\lim\limits_{x \to 0} f(x) =$ **$1$**

i. $\lim\limits_{x \to 4^+} f(x) =$ **$-\infty$**

2. **[4 points]** A full tank holds 2000 liters of water and is filled in one hour. The values in the table show the volume $V$ of water in the tank (in liters) after $t$ minutes.

| $t$ (minutes) | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $V$ (liters) | 0 | 200 | 500 | 900 | 1500 | 1800 | 2000 |

**Check differences** → 200  300  400  **600**  300  200
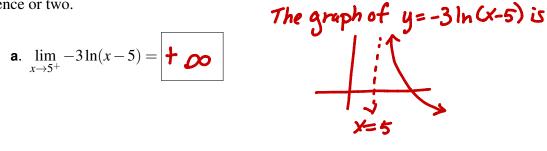
a. Find the average rate of change of the water in the tank in the first half of an hour. Include units in your answer.

$$\frac{900-0}{30-0} = \frac{90}{30} = 3 \text{ liters/min}$$

b. During what 10 minute interval was the average rate of change of the water the greatest (in magnitude)?

from $t=30$ to $t=40$   or   $[30,40]$

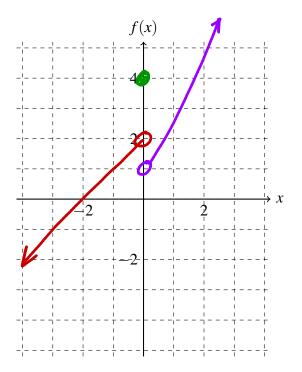3. **[6 points]** Compute the following infinite limits. For each limit, justify your answer with a sentence or two.

*The graph of $y = -3\ln(x-5)$ is*

a. $\displaystyle\lim_{x\to 5^+} -3\ln(x-5) = \boxed{+\infty}$

$x=5$

b. $\displaystyle\lim_{x\to\pi^-} \frac{x+2}{x-\pi} = \boxed{-\infty}$

*There is an asymptote at $x=\pi$.*
*For $x$ close to $\pi$, $x+2>0$.*
*For $x$ close to $\pi$ but larger, $x-\pi<0$.*

4. **[6 points]** On the axes below, sketch the graph of the function

$$f(x) = \begin{cases} 2+x & x<0 \\ 4 & x=0 \\ e^x & x>0. \end{cases}$$

Then compute, with brief justification, the requested values in the table.

$f(x)$

| Value | Justification |
|---|---|
| $f(0) = 4$ | by definition |
| $\displaystyle\lim_{x\to 0^-} f(x) =$   $2$ | We use formula $2+x$ |
| $\displaystyle\lim_{x\to 0} f(x) =$   DNE | There is a jump in the graph at $x=0$.   OR   The left side approaches 2 while the right side approaches 1. |