Math 251: Quiz 10 **Fall 2020**

Name: Solutions ________ / 20

20 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify unless asked, but show all work and use proper notation for full credit.

1. [5 points] Determine a function that satisfies the following constraints:

$$f''(x) = 12x^2 + \frac{6}{\sqrt{x}}, \quad f'(0) = 2, \quad f(1) = 4.$$

Clearly show your work.

$f'(x) = \int f''(x)\,dx = \int \left(12x^2 + \frac{6}{\sqrt{x}}\right)dx = 4x^3 + 12\sqrt{x} + \boxed{C_1}$

$f'(0) = 2 \Rightarrow f'(0) = 4\cdot 0 + 12\cdot 0 + C_1 = 2 \Rightarrow \boxed{C_1 = 2}$

$\boxed{f'(x) = 4x^3 + 12\sqrt{x} + 2}$

$f(x) = \int f'(x)\,dx = \int (4x^3 + 12\sqrt{x} + 2)\,dx = x^4 + 8x^{3/2} + 2x + \boxed{C_2}$

$f(1) = 4 \Rightarrow 1 + 8 + 2 + C_2 = 4 \Rightarrow \boxed{C_2 = -7}$

$\boxed{f(x) = x^4 + 8x^{3/2} + 2x - 7}$

2. [6 points] Compute the following integrals. Show your work.

a. $\displaystyle\int_0^{\pi/6} \frac{\sin(x)}{8} + x\,dx = \frac{1}{8}\int_0^{\frac{\pi}{6}} \sin(x)\,dx + \int_0^{\frac{\pi}{6}} x\,dx = \frac{1}{8}(-\cos x)\Big|_0^{\frac{\pi}{6}} + \frac{x^2}{2}\Big|_0^{\frac{\pi}{6}} =$

$= \frac{1}{8}\cdot\left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{8} + \frac{\pi^2}{72} = \boxed{\frac{1}{8} - \frac{\sqrt{3}}{16} + \frac{\pi^2}{72}}$

b. $\displaystyle\int \frac{3 - te^t}{t}\,dt = \int \frac{3}{t}\,dt - \int e^t\,dt = \boxed{3\ln|t| - e^t + C}$

3. [1 points] If $g(x) = \int_3^x \ln(t^2)\,dt$, find $g'(2)$.

By FTC part 1, $g'(x) = \ln(x^2)$

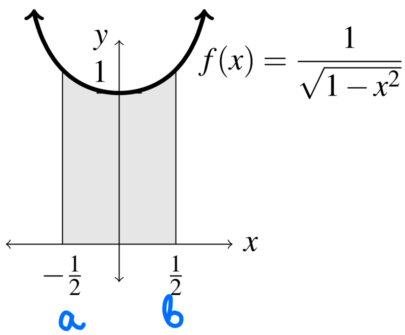
Then $\underline{g'(2) = \ln(2^2) = \ln(4)}$

4. [2 points] Find the derivative of the function $F(x) = \int_4^{\ln(x)} \tan(t)\sqrt{3t^5 - 2}\,dt$.

$F'(x) = \tan(\ln(x))\sqrt{3(\ln(x))^5 - 2} \cdot (\ln(x))' =$

$= \underline{\tan(\ln(x))\sqrt{3(\ln(x))^5 - 2} \cdot \frac{1}{x}}$

5. [4 points] Find the **exact** value of the area shaded below. The thick curve is $f(x) = \frac{1}{\sqrt{1-x^2}}$. Show your work and simplify your answer.



Use a definite integral:

$$A = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)\Big|_{-\frac{1}{2}}^{\frac{1}{2}} =$$

$$= \arcsin\left(\tfrac{1}{2}\right) + \arcsin\left(\tfrac{1}{2}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \boxed{\frac{\pi}{3}}$$

6. [2 points] Suppose $r(t)$ is the rate of change of the number of positive cases of COVID-19 in Alaska, measured in cases per month (computed on the last day of the month, say), where $t = 0$ is March 2020.

a. What does $\int_6^8 r(t)\,dt$ measure? Use complete sentences.

Let $N(t)$ be a number of positive cases of Covid-19

Then $r(t) = \frac{dN}{dt}$. Hence, by the Net Change Theorem we have

$\int_6^8 r(t)\,dt = N(8) - N(6)$ represents the change in number of pos. cases from September 2020 until November 2020

b. Is it possible for $\int_a^b r(t)\,dt$, where $a < b$ and $a, b \geq 0$, to be a negative number? Why or why not?

By the Net Change Theorem,

$\int_a^b r(t)dt = \underset{\substack{\uparrow\\ \text{number of cases at } t=b}}{N(b)} - \underset{\substack{\uparrow\\ \text{number of cases at } t=a}}{N(a)}$. Thus if the # of pos. cases decrease, then the difference $N(b) - N(a)$ is negative.

If the # of pos. cases increase, then the difference $N(b) - N(a)$ is positive.