Name: Solutions ______ / 20

There are 20 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [4 points]

a. Why is the following not a true statement? $\dfrac{x^2+x-6}{x-2} = x+3$

The statement is false since the domain of $f(x) = \frac{x^2+x-6}{x-2}$ is $(-\infty, 2)\cup(2,\infty)$ and the domain of $g(x) = x+3$ is $\mathbb{R}$.
In other words, for $x=2$ the left is undefined and the right is 5.

b. Nevertheless, explain why the following equation is correct. $\displaystyle\lim_{x\to 2}\frac{x^2+x-6}{x-2} = \lim_{x\to 2}(x+3)$

The limit does not care what happens precisely at $x=2$. The functions are the same for all other values.

2. [4 points] Compute the limit, if it exists. If the limit does not exist, explain why.

$$\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}. \quad = \frac{3-3}{0} = \frac{0}{0}$$

Need to do some algebra:

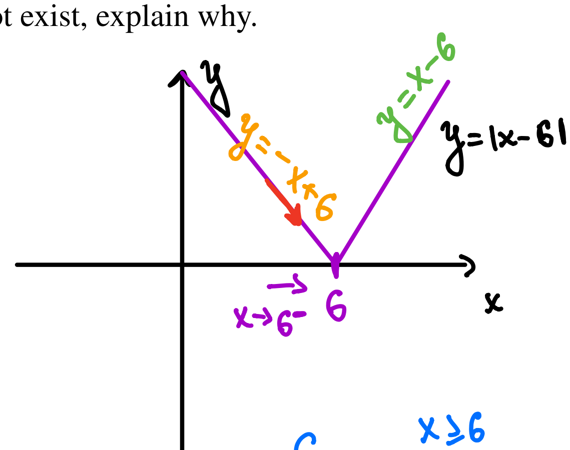
$$\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h} = \lim_{h\to 0}\frac{(\sqrt{9+h}-3)(\sqrt{9+h}+3)}{h(\sqrt{9+h}+3)} = \lim_{h\to 0}\frac{\cancel{9}+h-\cancel{9}}{h(\sqrt{9+h}+3)} =$$

$$= \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

3. [4 points] Compute the limit, if it exists. If the limit does not exist, explain why.

$$\lim_{x\to 6^-}\frac{2x-12}{|x-6|}$$

Have to deal at first with $f(x) = |x-6|$, when $x\to 6^-$



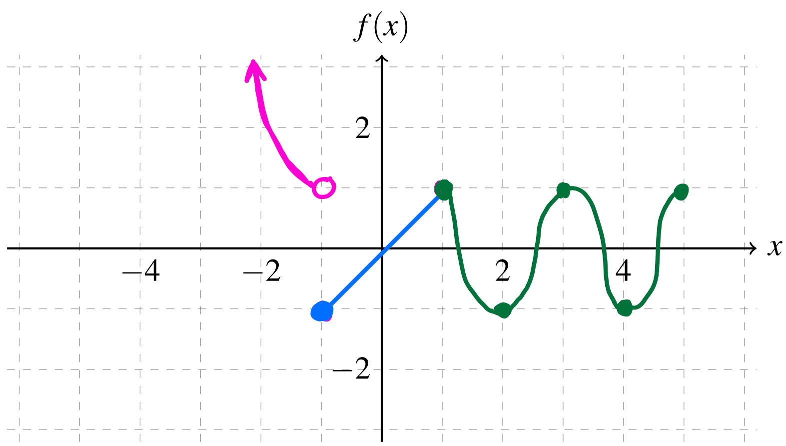
$$|x-6| = \begin{cases} x-6, & x-6\geq 0 \quad (x\geq 6) \\ -x+6, & x-6<0 \quad (x<6)\end{cases}$$

Hence

$$\lim_{x\to 6^-}\frac{2x-12}{-x+6} = \lim_{x\to 6^-}\frac{2\cancel{(x-6)}}{-\cancel{(x-6)}} = \boxed{-2}$$

4. [4 points] Consider the function $f(x) = \begin{cases} x^2 & x < -1 \\ x & -1 \leq x < 1 \\ -\cos(\pi x) & x \geq 1. \end{cases}$

a. In the diagram below, graph $f(x)$.



b. Determine whether or not $f(x)$ is continuous at $x = -1$ and explain your answer. You must use the <u>definition</u> of continuity in your explanation.

Def. Function $f(x)$ is continuous at $x=a$ if

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$$

We see that $\lim_{x\to -1^+} f(x) = -1 \neq \lim_{x\to -1^-} f(x) = 1$. Hence, $f(x)$ <u>is not</u> continuous at $x=1$.

5. [4 points] Use the Intermediate Value Theorem to justify the claim that there exists a number x in the interval $(0,-2)$ satisfying $xe^x = x^2 - 1$. Explain your answer.

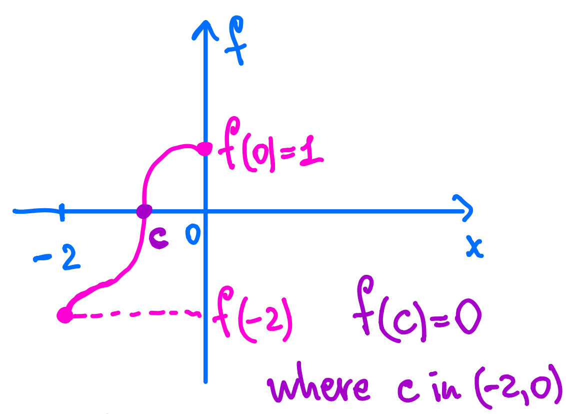
$xe^x - x^2 + 1 = 0$

Let $f(x) = xe^x - x^2 + 1$.

- $f(x)$ is continuous on $[0,-2]$ since it is a linear combination of continuous functions.

We check that

- $f(0) = 1 > 0$
- $f(-2) = -2\cdot\frac{1}{e^2} - 4 + 1 < 0$



Then there exists a number c in $(-2,0)$ such that $f(c)=0$, in other words, c is a solution of $c\cdot e^c = c^2 - 1$.