Name: Solutions ______ / 20

There are 20 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] For each function below, find its derivative. $\boxed{\text{You do not need to simplify your answer}}$.

a. $f(x) = 2\cos(x) - \sec(x)$

$f'(x) = (2\cos x)' - (\sec x)' = 2\cdot(-\sin x) - \sec x \tan x =$

$= -2\sin x - \sec x \tan x$

b. $r = \frac{3}{2}(\sin(\pi\theta))^3$

$r'(\theta) = \frac{3}{2}\cdot 3(\sin(\pi\theta))^2 \cdot \cos(\pi\theta)\cdot\pi = \frac{9\pi}{2}(\sin(\pi\theta))^2\cos(\pi\theta)$

c. $y = \tan\left(3x^2+2\right)$

$y'(x) = \sec^2(3x^2+2)\cdot 6x$

d. $g(x) = \underbrace{x^2e^x}_{f}\underbrace{(2-x^3)^5}_{h}$

$g'(x) = f'(x)h(x) + f(x)h'(x)$

$g'(x) = (2x\cdot e^x + x^2\cdot e^x)(2-x^3)^5 + x^2\cdot e^x\cdot 5(2-x^3)^4\cdot(-3x^2)$

e. $s(t) = \dfrac{\sin(t^{3/2})}{e^{2t}-t}$

$s'(t) = \dfrac{\cos(t^{3/2})\cdot\frac{3}{2}t^{1/2}(e^{2t}-t) - \sin(t^{3/2})\cdot(2e^{2t}-1)}{(e^{2t}-t)^2}$

2. [6 points] The displacement of a particle on a vibrating string is given by the equation

$$s = 8\cos\left(\pi t + \frac{\pi}{4}\right),$$

where t is measured in seconds and s is measured in centimeters.

a. Calculate the velocity and acceleration of the particle at any time t.

velocity: $S'(t) = -8\sin(\pi t + \frac{\pi}{4}) \cdot \pi$

acceleration: $S''(t) = V'(t) = -8\cos(\pi t + \frac{\pi}{4}) \cdot \pi^2$

b. Using the results from part (a), determine the position, velocity and acceleration of the particle at $t = 1$ second including **units**.

position at $t=1$: $S(1) = 8\cos(\pi \cdot 1 + \frac{\pi}{4}) = 8 \cdot \cos\frac{5\pi}{4} = \frac{-8\sqrt{2}}{2}$ (centimeters)

velocity at $t=1$: $V(1) = S'(1) = -8\sin(\pi \cdot 1 + \frac{\pi}{4}) \cdot \pi = -8\pi \sin\frac{5\pi}{4} = \frac{8\pi\sqrt{2}}{2}$ (cen/s)

acceleration at $t=1$: $a(1) = S''(1) = V'(1) = -8\pi^2 \cos\frac{5\pi}{4} = \frac{8\pi^2\sqrt{2}}{2}$ (cen/s^2)

3. [4 points] Find the equation of the tangent line to the curve $y = \frac{1}{\sqrt{1+4x}}$ at the point $(0, 1)$.

x_0 $f(x_0)$

TL equation:

$y = f'(x_0)(x - x_0) + f(x_0)$

We have $x_0 = 0$, $f(x_0) = 1$

$f'(x) = -\frac{1}{2}(1+4x)^{-3/2} \cdot 4$

$f'(0) = -\frac{1}{2} \cdot 4 \cdot 1 = -2$

Hence,

$y = -2(x-0) + 1 = -2x + 1$

$\boxed{y = -2x+1}$ TL equation