Name: Solutions ______ / 20

There are 20 points possible on this quiz. No aids (book, notes, etc.) are permitted. **You may use a calculator on this quiz to compute your final answer, but you must show all your work leading up to where you use your calculator and you must also show an exact answer.** Show all work for full credit.

1. [5 points] Strontium-90 has a half-life of 28 days. Suppose that a sample has a mass of 50 mg initially.

a. Find a formula for the mass remaining after t days.

$m(t) = m_0 e^{rt}$

$m(0) = 50 = m_0$

$m(28) = 25$

$m(28) = 25 = m_0 e^{r \cdot 28}$

$25 = 50 e^{r \cdot 28}$

$e^{r \cdot 28} = \frac{1}{2} \Rightarrow r \cdot 28 = \ln\left(\frac{1}{2}\right) \Rightarrow r = \frac{1}{28}\ln\left(\frac{1}{2}\right)$

Hence, $m(t) = 50 e^{\frac{1}{28}\ln\left(\frac{1}{2}\right)t} = 50 \cdot 2^{-\frac{t}{28}}$

b. How long would it take for the mass to decay to 10 mg?

$m(t) = 50 \cdot 2^{-\frac{t}{28}}$

$10 = 50 \cdot 2^{-\frac{t}{28}}$

$\frac{1}{5} = 2^{-\frac{t}{28}}$

$\frac{1}{5} = e^{\frac{1}{28}\ln\left(\frac{1}{2}\right)t}$

$\ln\left(\frac{1}{5}\right) = \frac{1}{28}\ln\left(\frac{1}{2}\right)t$

$t = \dfrac{28\ln\left(\frac{1}{5}\right)}{\ln\left(\frac{1}{2}\right)}$ (days)

2. [5 points] A medical researcher is studying the spread of the flu virus through a certain campus during the winter months. The model for the spread is described by

$$P(t) = \frac{240}{1 + 5e^{-t/7}},$$

where P represents the total number of infected students and t is the time, measured in days.

a. What is the infection rate after 7 days? That is, when $t = 7$ what is the rate of change of infections?

$P'(t) = -\dfrac{240}{(1+5e^{-t/7})^2} \cdot 5e^{-t/7} \cdot \left(-\frac{1}{7}\right)$

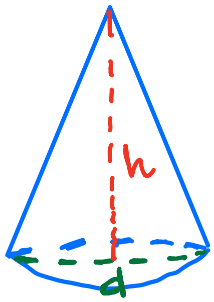
$P'(7) = -\dfrac{240}{(1+5e^{-1})^2} \cdot 5e^{-1}\left(-\frac{1}{7}\right) = \dfrac{240 \cdot 5 \cdot e}{7(e+5)^2}$ students/day

≈ 8 students/day

b. In the long term, how many students are infected?

$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \dfrac{240}{1+5e^{-t/7}} = \dfrac{240}{1} = 240$ (students)

3. [5 points] Sand is poured onto a surface at 15 cm^3/sec, forming a conical pile whose base diameter is always equal to its height. How fast is the height of the pile increasing when the pile is 3 cm high ? [Use the formula $V = \frac{1}{3}\pi r^2 h$ for computing the volume of the cone]



We have:

$\frac{dV}{dt} = 15 \text{ cm}^3/\text{sec}$

$d = h$

We want:

$\left.\frac{dh}{dt}\right|_{h=3\text{cm}} - ?$

Relation equation:

$V = \frac{1}{3}\pi r^2 h$

$d = 2r = h \Rightarrow r = \frac{h}{2}$

$V = \frac{1}{3}\pi \frac{h^3}{4} = \frac{\pi}{12} h^3$

Implicit differentiation:

$\frac{dV}{dt} = \frac{\pi}{12}\cdot 3h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{12}{\pi \cdot 3h^2}\cdot \frac{dV}{dt}$

$\left.\frac{dh}{dt}\right|_{h=3\text{cm}} = \frac{\cancel{12}^{\,4}}{\pi\cdot \cancel{27}_{\,3}}\cdot \cancel{15}^{\,5} = \frac{20}{3\pi} \left(\text{cm}/\text{sec}\right)$

4. [5 points]

a. Find the linear approximation (linearization) of the function $y = \sqrt{1-x}$ at $a = 0$.

$L(x) = f'(x_0)(x-x_0) + f(x_0)$

$x_0 = a = 0$

$f'(x) = \frac{-1}{2\sqrt{1-x}}$

$f'(0) = -\frac{1}{2}$

$f(0) = \sqrt{1-0} = 1$

$\boxed{L(x) = -\frac{1}{2}x + 1}$

b. Use linearization or differentials to **estimate** $\sqrt{0.9}$. Clearly show your work.

$\sqrt{0.9} = \sqrt{1-0.1} \approx L(0.1) = -\frac{1}{2}\cdot 0.1 + 1 = -\frac{1}{20} + 1 = \frac{19}{20} = 0.95$