Name: Solutions ______ / 20

There are 20 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [7 points] Consider the function $f(x) = \frac{x^4}{4} - 6x^2 + 16x + 1$ and note that $f'(x) = 16 - 12x + x^3 = (-2+x)^2(4+x)$. Determine the intervals where f is increasing and decreasing, where f is concave up and concave down, and list all local maxima, minima, and inflection points. If none, indicate that.

You must clearly show your work to receive credit. Consider using some words.

1. Critical points: $f'(x)=0 \Rightarrow (-2+x)^2(4+x)=0 \Rightarrow \boxed{x=-4}$ and $\boxed{x=2}$

x	$x<-4$	$x=-4$	$-4<x<2$	$x=2$	$x>2$
$f'(x)$	$-$	0	$+$	0	$+$
$f(x)$	$\searrow$	loc. min	$\nearrow$		$\nearrow$

2. Concavity intervals: $f''(x) = 3x^2 - 12 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

x	$x<-4$	$x=-4$	$-4<x<-2$	$x=-2$	$-2<x<2$	$x=2$	$x>2$
$f''(x)$	$+$	$+$	$+$	0	$-$	0	$+$
$f(x)$	$\cup$	$\cup$	$\cup$	inflec. point	$\cap$	inflec. point	$\cup$

$f(x)$ is increasing on $(-4,2)\cup(2,\infty)$ $f(x)$ is decreasing $(-\infty,-4)$

local max at $x =$ none local min at $x =$ -4

$f(x)$ is concave up on $(-\infty,-2)\cup(2,\infty)$ $f(x)$ is concave down $(-2,2)$

inflection point(s) at $x =$ $-2, 2$

2. [3 points] Compute the following limit. Show all your work, including stating the **indeterminate type** of the limit, if relevant. If you use L'Hospital's rule, you MUST indicate where you are doing so.

$$\lim_{t\to 0} \frac{(t-1)^2 - e^{3t}}{t} = \frac{0}{0}$$

Thus we have $\frac{0}{0}$ indeterminate form which requires us to use a L'H rule.

$$\lim_{t\to 0} \frac{(t-1)^2 - e^{3t}}{t} \overset{L'H}{=} \lim_{t\to 0} \frac{2(t-1) - 3e^{3t}}{1} = \frac{-2-3}{1} = \boxed{-5}$$

3. [3 points] Compute the following limit. Show all your work, including stating the **indeterminate type** of the limit, if relevant. If you use L'Hospital's rule, you $\boxed{\text{MUST}}$ indicate where you are doing so.

$\lim_{x\to\frac{\pi}{2}} \cos(x)\sec(3x)$ $= 0\cdot\infty$ indeterminate form

$$\lim_{x\to\frac{\pi}{2}} \cos x\cdot\sec(3x) = \lim_{x\to\frac{\pi}{2}} \frac{\cos x \; 0}{\frac{1}{\sec(3x)} \; 0} \overset{L'H}{=} \lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{-\sin(3x)\cdot 3} = \frac{-1}{-1\cdot(-1)\cdot 3} = \boxed{-\frac{1}{3}}$$

$$\frac{1}{\sec(3x)} = \cos(3x)$$

4. [7 points] Suppose you know that $q(x)$ is a function with the following properties:

- The domain of $q(x)$ is $(-\infty,-1)\cup(-1,\infty)$
- $\lim_{x\to-\infty} q(x) = 2$ and $\lim_{x\to\infty} q(x) = \infty$
- $x=-1$ is a vertical asymptote
- $q(0) = 1$

Furthermore, you know the following information about the signs of the first and second derivatives:

x	$x<-3$	-3	$-3<x<-1$	-1	$-1<x<4$	4	$x>4$
sign of q'	$+$	0	$-$	DNE	$+$	0	$+$

x	$x<-5$	-5	$-5<x<-1$	-1	$-1<x<4$	4	$x>4$
sign of q''	$+$	0	$-$	DNE	$-$	0	$+$

$\boxed{\text{Sketch a graph}}$ of a function with all these properties. **Clearly label** and **mark the x-coordinates on the x-axis** of all local maxima, minima and all inflection points, and **label** all asymptotes with their equations.