Name: Solutions _____ _____ / 20

20 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify unless asked, but show all work and use proper notation for full credit.

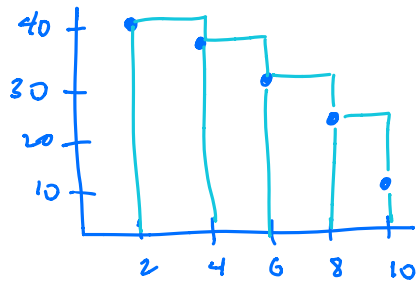
1. [5 points] The following table gives the velocity (in **m/s**) of an object at time t (in **seconds**).

t (in seconds)	2	4	6	8	10
$v(t)$ (in m/s)	40	38	32	25	10

Estimate the distance traveled between $t = 2$ and $t = 10$ using LEFT-HAND rectangles.

(a) Sketch a graph showing how you are estimating the distance traveled.

(b) Set up, but DO NOT COMPUTE, a calculation determining the distance traveled. (You can do the arithmetic if you like, but we don't need you to.)



Distance $= 2(40) + 2(38) + 2(30) + 2(25)$

(c) What are the units associated with your computation? meters

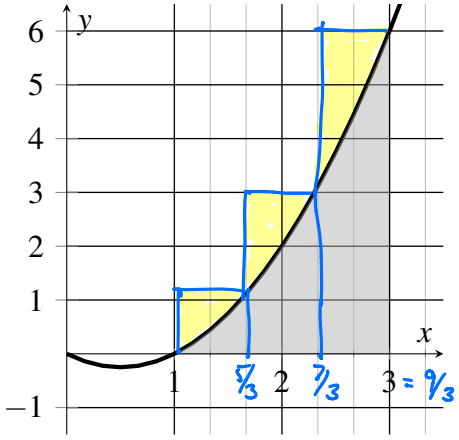
2. [5 points] We want to estimate the area (shaded in gray) under the graph of $f(x) = x^2 - x$ from 1 to 3 using the areas of **three rectangles** of equal width, where the heights of the rectangles are determined by the height of the curve at right-hand endpoints.

(a) Width of each rectangle = $2/3$

(b) DRAW the rectangles on the graph.

(c) Set up, but DO NOT EVALUATE, a computation to determine the area of the rectangles. Your computation should not include the symbols "$f(x)$".

$\frac{2}{3}\left(\left(\frac{5}{3}\right)^2 - \left(\frac{5}{3}\right)\right) + \frac{2}{3}\left(\left(\frac{7}{3}\right)^2 - \left(\frac{7}{3}\right)\right) + \frac{2}{3}\left(3^2 - 3\right)$



(d) Does your computation overestimate or underestimate the actual area, and why?

Overestimation — the rectangles include area that is not in the gray shaded area

3. [2 points] Given that f is the function whose graph is shown below and $g(x) = \int_0^x f(t)\,dt$, find the following. Show some work for possible partial credit.

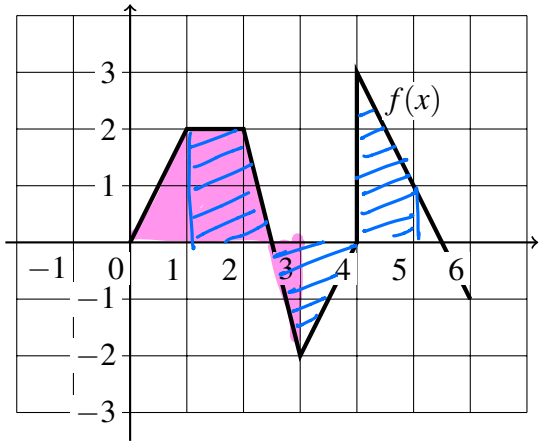
a. $g(3) =$ 3

the pink shaded area

b. $\int_1^5 f(t)\,dt =$ 3

the blue /// area

$= 2 + 0 - 1 + 2$



4. [5 points] [Fill in the blank] If $\int_1^5 f(x)\,dx = 7$, $\int_{-3}^1 g(x)\,dx = 12$ and $\int_1^5 g(x)\,dx = 13$, compute the following quantities or state that it cannot be evaluated from the given information:

a. $\int_1^1 f(x)\,dx =$ 0

b. $\int_5^1 4f(x)\,dx = -4\int_1^5 f(x)\,dx = -28$

c. $\int_{-3}^5 g(x)\,dx = \int_{-3}^1 g(x)\,dx + \int_1^5 g(x)\,dx = 12 + 13 = 25$

d. $\int_{-3}^1 [4g(x) - 10]\,dx = 4\int_{-3}^1 g(x)\,dx - \int_{-3}^1 10\,dx = 4(12) - 10(1-(-3)) = 48 - 40 = 8$

e. $\int_1^5 [5f(x) + 3g(x)]\,dx = 5\int_1^5 f(x)\,dx + 3\int_1^5 g(x)\,dx = 5(7) + 3(13) = 35 + 39 = 74$

5. [3 points] Evaluate the integral $\int_0^7 |x-3|\,dx$ by interpreting it in terms of area. Justify your answer by sketching a graph.

$\int_0^7 |x-3|\,dx = \frac{1}{2}(3)(3) + \frac{1}{2}(4)(4)$

$= \frac{1}{2}(9+16) = \frac{25}{2}$