Name: Solutions ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[3 points]** If $F(x) = \int_3^x t^5 \sin(t+1)\,dt$, find $F'(x)$.

$$F'(x) = x^5 \sin(x+1)$$

2. **[4 points]** Let $Q(x) = \int_0^x (t+1)\,dt$.

a. Find $Q(2)$.

$$Q(2) = \int_0^2 (t+1)\,dt = \frac{1}{2}t^2 + t\Big]_0^2 = \left(\frac{1}{2}\cdot 2^2 + 2\right) - \left(\frac{1}{2}0^2 + 0\right)$$
$$= \frac{4}{2} + 2 = 4$$

b. Find $Q'(2)$.

(2 ways to solve this)

Easiest:

$$Q'(2) = 2+1 = 3$$

Alternate: $Q(x) = \int_0^x (t+1)\,dt = \frac{1}{2}t^2 + t\Big]_0^x + Q(0)$

$$= \frac{1}{2}x^2 + x + Q(0).$$

So $Q'(x) = x+1$. So $Q'(2) = 2+1 = 3$

3. **[4 points]** Evaluate the definite integral $\int_{-2}^{2} (x^3 + 3x^2 - 5x)\,dx$.

(2 ways.)

Easiest: Observe that x^3 and $-5x$ are odd, so $\int_{-2}^{2} (x^3 - 5x)\,dx = 0$.

$$\int_{-2}^{2} (x^3 + 3x^2 - 5x)\,dx = \int_{-2}^{2} 3x^2\,dx = x^3\Big]_{-2}^{2}$$
$$= 2^3 - (-2)^3 = 8 + 8 = 16$$

Alternate: $= \frac{1}{4}x^4 + x^3 - \frac{5}{2}x^2\Big]_{-2}^{2}$

$$= \left(\frac{1}{4}(2^4) + 2^3 - \frac{5}{2}2^2\right) - \left(\frac{1}{4}(-2)^4 + (-2)^3 - \frac{5}{2}(-2)^2\right)$$
$$= (4 + 8 - 10) - (4 - 8 - 10) = 8 + 8 = 16$$

4. [6 points] Assume height of balloon is changing at rate of $r(t) = t - 2\cos(t)$ where t is measured in minutes and $r(t)$ is measured in feet per minute starting at time $t = 0$.

a. Evaluate $\int_0^\pi r(t)\,dt$ $= \int_0^\pi t - 2\cos(t)\,dt = \frac{1}{2}t^2 - 2\sin(t)\Big]_0^\pi$

$= \left(\frac{1}{2}\pi^2 - 2\sin(\pi)\right) - \left(\frac{1}{2}0^2 - 2\sin(0)\right)$

$= \frac{1}{2}\pi^2$ feet.

b. Interpret the meaning of the calculation from part (a). Include units in your answer.

The net change in the height of the balloon in the first π minutes was $\pi^2/2$ feet. (Alternate answer: At $t = \pi$ minutes, the ballooon was $\pi^2/2$ feet higher than its height at $t=0$ minutes.)

5. [8 points] Use the method of substitution to evaluate the integrals below.

a. $\int x^2(5 - x^3)^8\,dx$ $= \frac{-1}{3}\int u^8\,du = -\frac{1}{3}\cdot\frac{1}{9}u^9 + C$

let $u = 5 - x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$= -\frac{1}{27}(5 - x^3)^9 + C$

b. $\int \theta^{-2}\cos(\theta^{-1})\,d\theta$ $= -\int \cos(u)\,du = -\sin(u) + C$

let $u = \theta^{-1}$

$du = -\theta^{-2}\,d\theta$

$-du = \theta^{-2}\,d\theta$

$= -\sin(\theta^{-1}) + C$