Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[16 points]** (4 pts each; 2 pts for answer, 2 pts for work) Evaluate the following limits. Give the most complete answer; if the limit is infinite, indicate that with ∞ or $-\infty$. If a value does not exist, write DNE.

Substitution doesn't work; Factor!

a. $\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6} = \frac{2^2-4}{2^2-10+6} = \frac{0}{0}$

$= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(x-3)} = \lim_{x\to 2} \frac{x+2}{x-3} = \frac{2+2}{2-3} = \frac{4}{-1} = -4$

Sub. doesn't work. Do algebra!

b. $\lim_{h\to 0} \frac{\frac{3}{2}-\frac{3}{2+h}}{h} = \frac{\frac{3}{2}-\frac{3}{2}}{0} = \frac{0}{0}$

$= \lim_{h\to 0} \left(\frac{1}{h}\right)\left(\frac{3(2+h)-3(2)}{2(2+h)}\right) = \lim_{h\to 0} \left(\frac{1}{h}\right)\left(\frac{6+3h-6}{2(2+h)}\right) = \lim_{h\to 0} \frac{3h}{(h)(2)(2+h)}$

$= \lim_{h\to 0} \frac{3}{2(2+h)} = \frac{3}{2(2+0)} = \frac{3}{4}$

c. **Make sure to give some justification for your answer here.** $\lim_{t\to -3^+} \frac{5+t}{t^2+3t} = \frac{8}{9-9} = \frac{8}{0}$ This limit is infinite.

Figure out the sign:

$\lim_{t\to -3^+} \left(\frac{5+t}{t}\right)\left(\frac{1}{t+3}\right) = -\infty$

$\frac{5-3}{-3} = \frac{-2}{3}$ $\quad +\infty$

use $(-)(+)=-$

d. Given $\lim_{x\to 5} f(x) = 8$ and $\lim_{x\to 5} g(x) = -10$, evaluate $\lim_{x\to 5} \frac{3f(x)-x}{(g(x))^2}$. $= \frac{3(8)-5}{(-10)^2}$

$= \frac{24-5}{100} = \frac{19}{100}$

2. [4 points] Does the equation $x - \sin(\pi x) - 3 = 0$ have a solution on the interval from $x = 0$ to $x = 5$? Use the Intermediate Value Theorem to justify your answer.

Answer: Yes. The equation does have a solution.

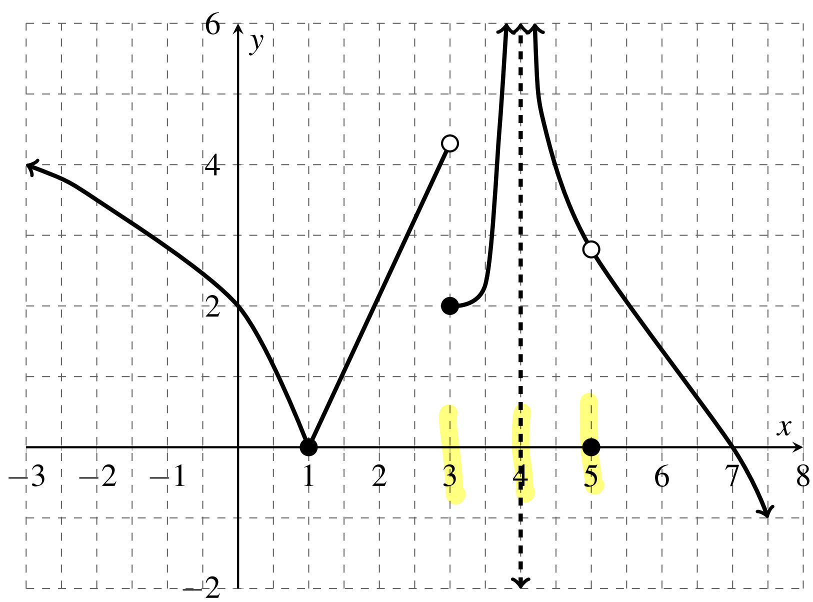
Let $f(x) = x - \sin(\pi x) - 3$, a continuous function.

$f(0) = 0 - \sin(0) - 3 = -3 < 0$.

$f(5) = 5 - \sin(5\pi) - 3 = 2 > 0$.

The IVThm tells us there is a c-value in the interval $(0,5)$ such that $f(c) = 0$. So the equation has a solution.

3. [5 points] Consider the graph of the function $y = H(x)$ shown in the graph below.



a. List all x-values for which the function $H(x)$ fails to be continuous.

$x = 3, 4, 5$

b. Label the values above as removable or nonremovable.

removable: $x = 5$, nonremovable: $x = 3, 4$