Name: ____________________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[4 points]** The function $C(y) = \frac{18(1+y)}{2y+5}$ models a herd of caribou where C is the number of caribou in hundreds and y is measured in years starting in the year 2000.

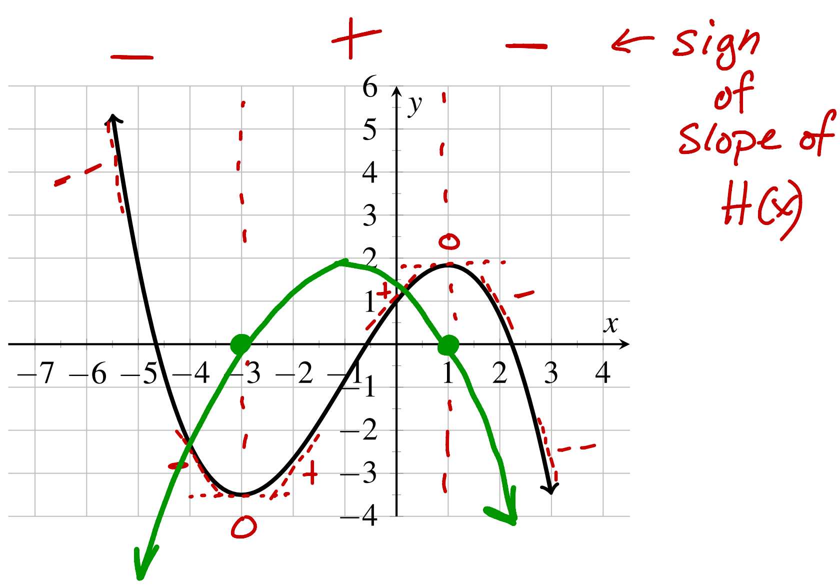
a. Observe that $C(10) = 7.92$. Interpret this fact in the context of the problem. To earn full credit your answer should be a complete sentence and must include units.

In 2010, there were 792 caribou.

b. It can be shown that $C'(10) = 0.0864$. Interpret this fact in the context of the problem. To earn full credit your answer should be a complete sentence and must include units.

In 2010, the population of caribou was increasing at a rate of 8.64 caribou per year.

2. **[4 points]** The function $y = H(x)$ is graphed below. Sketch the graph of $H'(x)$ on the same set of axes.



3. [9 points] Find the derivative of $f(x) = 3\sqrt{x}$ **using the limit definition of the derivative.** No credit will be given for an answer that uses the power rule.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3\sqrt{x+h}-3\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{3\sqrt{x+h}-3\sqrt{x}}{h} \cdot \frac{(3\sqrt{x+h}+3\sqrt{x})}{(3\sqrt{x+h}+3\sqrt{x})} = \lim_{h\to 0} \frac{9(x+h)-9x}{h(3\sqrt{x+h}+3\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{9x+9h-9x}{h(3\sqrt{x+h}+3\sqrt{x})} = \lim_{h\to 0} \frac{9}{3\sqrt{x+h}+3\sqrt{x}} = \frac{9}{6\sqrt{x}} = \frac{3}{2\sqrt{x}}$$

Check myself: $y = 3x^{1/2}$ $\quad y' = \frac{3}{2}x^{-1/2}$ ✓

4. [8 points] For each function below, find its derivative. You may use any method you like. You do not have to simplify your answer.

a. $f(x) = \frac{x^3+x-\pi^2}{3}$

Simplify first:

$f(x) = \frac{1}{3}x^3 + \frac{1}{3}x - \frac{\pi^2}{3}$

$\boxed{f'(x) = x^2 + \frac{1}{3}}$

Quotient Rule

$$f'(x) = \frac{3(3x^2+1) - (x^3+x-\pi^2)(0)}{3^2}$$

$$= \frac{3(3x^2+1)}{9} = \frac{1}{3}(3x^2+1) = \boxed{x^2 + \frac{1}{3}}$$

b. $g(x) = x\left(\frac{1}{x^2} + \frac{1}{x}\right) = x(x^{-2}+x^{-1})$

Simplify first:

$g(x) = x^{-1} + 1$

$g'(x) = -x^{-2}$

product rule

$$g'(x) = 1(x^{-2}+x^{-1}) + x(-2x^{-3} - x^{-2})$$

$$= x^{-2} + x^{-1} - 2x^{-2} - x^{-1}$$

$$= -x^{-2}$$