Name: Solutions _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[16 points]** Find $\frac{dy}{dx}$. You do not have to simplify

a. $y = \cos^{-1}(\sqrt{x})$ (§3.7 # 279) $= \cos^{-1}(x^{1/2})$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \left(\frac{1}{2}x^{-1/2}\right) = \boxed{\frac{-1}{2\sqrt{x}\sqrt{1-x}}}$$

b. $y = \left(x + \sin^{-1}(x)\right)^5$ (§3.7 # 283)

$$\frac{dy}{dx} = \boxed{5\left(x+\sin^{-1}(x)\right)^4 \cdot \left(1 + \frac{1}{\sqrt{1-x^2}}\right)}$$

c. $y = e^{2x}\cos(x)$ (§3.9 #331,331,333)

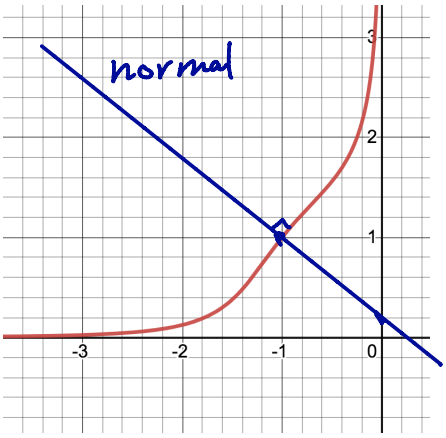
$$\frac{dy}{dx} = \left(2e^{2x}\right)\left(\cos(x)\right) + \left(e^{2x}\right)\left(-\sin(x)\right)$$

$$= \boxed{e^{2x}\left(2\cos(x) - \sin(x)\right)}$$

d. $y = \ln(8x+1)$ (§3.9 # 340, 341)

$$\frac{dy}{dx} = \frac{1}{8x+1} \cdot 8 = \boxed{\frac{8}{8x+1}}$$

2. [5 points] The graph of $x^4y - xy^3 = 2$ is sketched below. (§3.8 # 313, 317)



(a) Find dy/dx.

$$4x^3y + x^4\frac{dy}{dx} - y^3 - x\cdot 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(x^4 - 3xy^2\right) = y^3 - 4x^3y$$

$$\boxed{\frac{dy}{dx} = \frac{y^3 - 4x^3y}{x^4 - 3xy^2}}$$

(b) Write an equation for the line **normal** to the curve at the point $(-1, +1)$ and **sketch** the line on the graph.

Point $(-1,+1)$

Slope of tangent $= m = \left.\frac{dy}{dx}\right|_{(-1,+1)} = \frac{1^3 - 4(-1)^3(1)}{(-1)^4 - 3(-1)(1)^2} = \frac{1+4}{1+3} = \frac{5}{4}$

slope of normal : $-\frac{4}{5}$

line : $y - 1 = -\frac{4}{5}(x+1)$ or

$$\boxed{y = -\frac{4}{5}x + \frac{1}{5}}$$

3. [4 points] Find the derivative of $y = (x)^{\sin(x)}$. (Recall that you will have to use **logarithmic differentiation**. §3.9 # 347,348 and §3.8)

$$\ln(y) = \ln\left(x^{\sin(x)}\right) = \sin(x)\ln(x)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \cos(x)\ln(x) + \sin(x)\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = y\left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right)$$

$$\frac{dy}{dx} = \left(x^{\sin(x)}\right)\left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right)$$