Name: _____ _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[8 points]** Follow the steps below to solve a related rates problem.

   a. Assume the base, b, of a triangle is growing at a rate of 2 feet per minute and the height, h, of the triangle is shrinking at a rate of 4 feet per minute when the base is 10 feet long and height is 15 feet long. Using this information, identify values for h, b, dh/dt and db/dt.

   $b = 10\text{ ft}$ $\quad db/dt = 2\text{ ft/min}$

   $h = 15\text{ ft}$ $\quad dh/dt = -4\text{ ft/min}$

   b. The area of a triangle is given by the formula $A = \frac{1}{2}bh$ where b is the length of the base of the triangle and h is its height. Take the derivative of the above equation implicitly with respect to time.

   $A = \frac{1}{2} b \cdot h$ → product rule!

   $$\frac{dA}{dt} = \frac{1}{2}\left[b \cdot \frac{dh}{dt} + \frac{db}{dt} \cdot h\right]$$

   c. Use the above information to determine the rate of change of the area of the triangle. Include units.

   $$\frac{dA}{dt} = \frac{1}{2}\left(10 \cdot (-4) + 2(15)\right) = \frac{1}{2}(-40+30) = -5\ \text{ft}^2/\text{min}$$

   d. Is the area increasing or decreasing at this instant? Justify your answer.

   Decreasing. $\frac{dA}{dt} < 0$.

2. [8 points] Let $f(x) = x^4$.

a. Find the linear approximation, $L(x)$, of $f(x)$ at $x = 2$.

$f(2) = 2^4 = 16$

$f'(x) = 4x^3$

$f'(2) = 4(2^3)$

$= 32$

$\boxed{L(x) = 16 + 32(x-2)}$

b. Use the linear approximation to estimate $(1.8)^4$. Your answer here must be in the form of a simplified fraction. or decimal.

$(1.8)^4 \approx L(1.8) = 16 + 32(1.8-2) = 16 + (32)(-0.2)$

$= 16 - 6.4$

$= \boxed{9.6} = \frac{96}{10} = \boxed{\frac{48}{5}}$

aside:

$32(0.2) = 6.4$

3. [9 points] Let $g(x) = 3x^4 - 4x^3$.

a. Find all critical points of $g(x)$.

$g'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

aside: $\frac{1}{16}$ $\frac{3}{48}$

$\boxed{\text{c.points: } x=0 \text{ and } x=1}$

b. Determine the **absolute** minimum and **absolute** maximum of $g(x)$ on the interval $[-1, 2]$. Make sure to show your work.

end points

critical points

make a table

x	-1	2	0	1
$y = g(x)$	$g(-1) = 3+4 = 7$	$g(2) = 48-32 = 16$	0	$g(1) = 3-4 = -1$
		↑ largest		↑ smallest

answer:

absolute maximum: 16

absolute minimum: -1