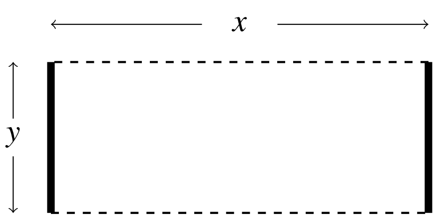
Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[9 points]** (Optimization) You need to construct a rectangular fence that encloses an area of 300 square feet. The two vertical sides (drawn solid below) will be made of material that costs \$5 per foot while the material for the horizontal sides (drawn dashed below) costs \$2 per foot. Determine the dimensions of the least expensive fence. Make sure you explicitly address the items below.



$A = xy = 300\text{ ft}^2 \qquad y = 300x^{-1}$

$C = 2x \cdot 2 + 2y \cdot 5 = 2 \cdot x + 10y$

a. Explicitly state the quantity you want to maximize or minimize. minimize cost

b. Identify the domain of your function.

c. Identify your answer. (Note: Your answer may not be an integer.)

d. Justify that your answer is correct. That is, use Calculus to show that your answer indeed does represent a maximum or minimum.

b

$C(x) = 4x + 10(300x^{-1}) = 4x + 1000x^{-1}$; domain: $(0,\infty)$

$C'(x) = 4 - 1000x^{-2} = 0$

$4 = \frac{1000}{x^2}$ or $x^2 = 250$ or $x = +\sqrt{250} = 5\sqrt{10}$.

d Justification: $C' < 0$ when $x < 5\sqrt{10}$ and
$C' > 0$ when $x > 5\sqrt{10}$

So C has a local min at $x = 5\sqrt{10}$. It is an absolute maximum because it is the only critical point.

c Answer: $x = 5\sqrt{10}\text{ ft}$ and $y = \frac{300}{5\sqrt{10}} = \frac{60}{\sqrt{10}} = 6\sqrt{10}\text{ ft}$

2. [8 points] Evaluate the following limits. Before an application of L'Hôpital's Rule, you must indicate the form of the limit (0/0 or ∞/∞).

a. $\lim_{x \to 1} \frac{x^{14}-1}{x^5-1}$ $\overset{H}{=} \lim_{x \to 1} \frac{14x^{13}}{5x^4} = \frac{14}{5}$

form $\frac{0}{0}$

b. $\lim_{x \to \infty} \left(1+\frac{2}{x}\right)^x$ $= e^2$

$\lim_{x \to \infty} x \ln(1+2x^{-1}) = \lim_{x \to \infty} \frac{\ln(1+2x^{-1})}{x^{-1}} \overset{H}{=} \lim_{x \to \infty} \frac{\frac{1}{1+2x^{-1}} \cdot -2x^{-2}}{-x^{-2}}$

form $\frac{0}{0}$ $= \lim_{x \to \infty} \frac{2}{1+2x^{-1}} = 2$

3. [8 points] Evaluate the following indefinite integrals.

a. $\int \left(2+x+\frac{1}{x^2}\right) dx$

$= \int (2+x+x^{-2})dx = 2x + \frac{1}{2}x^2 - x^{-1} + C$

b. $\int (\sec(x)\tan(x)+e^x)\, dx$ $= \sec x + e^x + C$