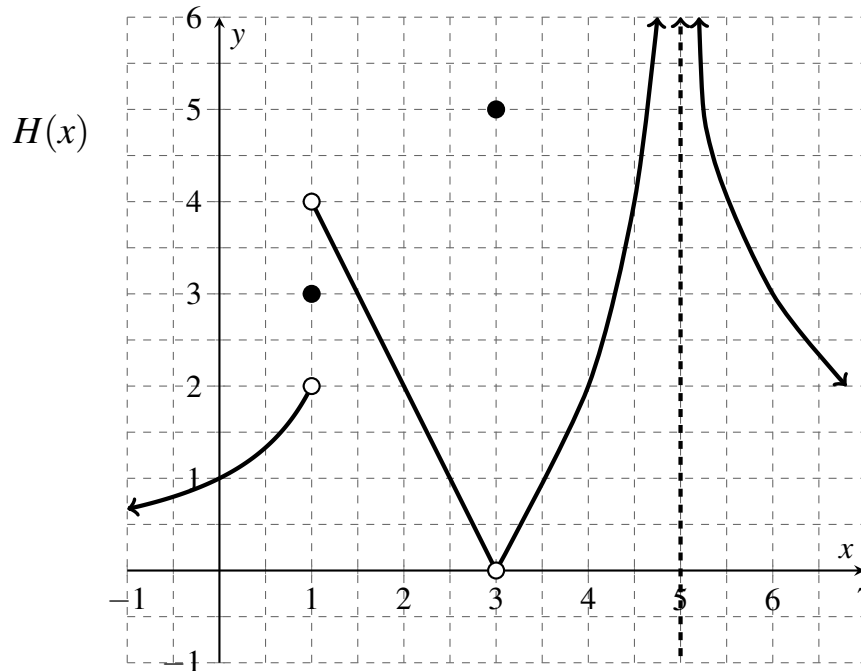


Name: Solutions

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (10 points) Use the graph of the function $H(x)$ (drawn below) to answer the questions. Assume $H(x)$ has a vertical asymptote at $x = 5$. For each problem below, give the most complete answer; if the limit is infinite, indicate that with ∞ or $-\infty$.



(a) $H(0) = \underline{1}$

(b) $H(1) = \underline{3}$

(c) $H(3) = \underline{5}$

(d) $\lim_{x \rightarrow 1^-} H(x) = \underline{2}$

(e) $\lim_{x \rightarrow 1^+} H(x) = \underline{4}$

(f) $\lim_{x \rightarrow 1} H(x) = \underline{DNE}$

(g) $\lim_{x \rightarrow 0} H(x) = \underline{1}$

(h) $\lim_{x \rightarrow 3} H(x) = \underline{0}$

(i) $\lim_{x \rightarrow 5} H(x) = \underline{\infty}$

- (j) List all x -values for which the function $H(x)$ fails to be continuous.

$x = 1, 3, 5$

(Note: DNE is true; but not the most complete answer.)

2. (12 points) Evaluate the following limits. Show your work to earn full credit.

$$(a) \lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{x+1} = \lim_{x \rightarrow -1} x-1 = -1-1 = \boxed{-2}$$

↑
 plugin.
 get 0.
 Do algebra.

$$(b) \lim_{x \rightarrow 0} \frac{\frac{2}{3+x} - \frac{2}{3}}{x} = \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{2}{3+x} - \frac{2}{3} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{2 \cdot 3 - 2(3+x)}{(3+x)(3)} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{-2x}{3(3+x)} \right)$$

$6 - 6 - 2x = -2x$

$$= \lim_{x \rightarrow 0} \frac{-2}{3(3+x)} = \frac{-2}{3(3+0)} = \frac{-2}{9}$$

↑
 plug in
 $x=0$.
 Get 0.
 Do algebra.

$$(c) \lim_{x \rightarrow 5^+} \frac{1 + \sqrt{x+4}}{5-x} = \frac{1 + \sqrt{9}}{0} = \frac{4}{0} \leftarrow \text{So limit is } \neq \infty!$$

Answer:

$$\lim_{x \rightarrow 5^+} \frac{1 + \sqrt{x+4}}{5-x} = -\infty$$

Work.

as $x \rightarrow 5^+$, $1 + \sqrt{x+4} = 4 > 0$
 $5-x \rightarrow 0^-$.

[So, numerator is positive, but denominator is negative.]

(d) If $\lim_{x \rightarrow 2} f(x) = 7$, find $\lim_{x \rightarrow 2} (5 - 2x + 3f(x)) =$

$$= \lim_{x \rightarrow 2} 5 + \lim_{x \rightarrow 2} (-2x) + 3 \lim_{x \rightarrow 2} f(x)$$

$$= 5 + (-4) + 3(7) = 22$$

3. (3 points) Pick k such that $f(x)$ is continuous if $f(x) = \begin{cases} x^2 & x \leq 2 \\ 3x+k & x > 2 \end{cases}$.

We need $x^2 = 3x+k$ when $x=2$.

$$\text{So } 2^2 = 3 \cdot 2 + k$$

$$\text{So } \boxed{k = -2}$$