Name: _____ Solutions _____   _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [2 points] State the definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2. [8 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{5}{x+1}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{5}{x+h+1} - \frac{5}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{5(x+1) - 5(x+h+1)}{(x+h+1)(x+1)} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \left(\frac{5x + 5 - 5x - 5h - 5}{(x+h+1)(x+1)} \right) = \lim_{h \to 0} \frac{1}{h} \left(\frac{-5h}{(x+h+1)(x+1)} \right)$$

$$= \lim_{h \to 0} \frac{-5}{(x+h+1)(x+1)} = \frac{-5}{(x+0+1)(x+1)} = \frac{-5}{(x+1)^2}$$

the derivative of

3. [3 points] Use the Quotient Rule to find $f(x) = \frac{\sin(x)}{x+1}$. (You do not need to simplify.)

$$f'(x) = \frac{(x+1)(\cos(x)) - (\sin x)(1)}{(x+1)^2} = \frac{(x+1)\cos(x) - \sin(x)}{(x+1)^2}$$

4. [6 points] Find the derivative for each function below.

a. $y = 2x^{4.1} - x + \pi^2$

$$y' = 2(4.1)x^{3.1} - 1 = 8.2x^{3.1} - 1$$

b. $y = x\cos(x)$

$$y' = 1 \cdot \cos(x) + x(-\sin(x))$$

5. [6 points] Assume $C(q)$ is the cost, in dollars, of manufacturing q widgets.

a. Using a complete sentence (or sentences), interpret the meaning of $C(50) = 1120$.

It costs $1120 to manufacture 50 widgets.

b. What are the units of $C'(q)$?

$$\frac{\Delta C}{\Delta q} = \frac{\$}{\text{widgets}} \quad \text{or} \quad \text{dollars per widget.}$$

c. Using a complete sentence (or sentences), interpret the meaning of $C'(50) = 15$.

Manufacturing costs are increasing at a rate of $15/widget when 50 widgets are produced.