Name: ___Solutions___   _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[9 points]** (Related Rate Problem) The radius of a cylinder is increasing at a rate of $2\,cm/s$ while the volume of the cylinder is decreasing at a rate of $25\pi\,cm^3/s$. How fast is the height of the cylinder changing when the radius is $5\,cm$ and the height is $10\,cm$? Interpret your answer using a complete sentence. Units should be included in your answer.

$$\frac{dr}{dt} = 2, \quad \frac{dV}{dt} = 25\pi$$

We want $\frac{dh}{dt}$ when $r=5$ and $h=10$.

$$V = \pi r^2 h$$

Take derivative implicitly w.r.t. time.
Use product rule on $(r^2)(h)$.

$$\frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}\cdot h + r^2\cdot\frac{dh}{dt}\right)$$

Plug in

$$25\pi = \pi\left(2(5)(2)(10) + 5^2\cdot\frac{dh}{dt}\right)$$

$$25 = 200 + 25\frac{dh}{dt}$$

Solve for $\frac{dh}{dt}$

$$\frac{25-200}{25} = \frac{dh}{dt}$$

So $\frac{dh}{dt} = \frac{-175}{25} = -7\,cm/s$

The height of the cylinder is decreasing at a rate of $7\,cm/s$.

2. [8 points] (Linear Approximation and Differentials) Let $h(x) = 5 - 2\sin(x-3)$.

a. Find the differential of $h(x)$.

$$\frac{dh}{dx} = -2\cos(x-3)$$

$$\boxed{\text{differential: } dh = -2\cos(x-3)\,dx}$$

b. Find the differential of $h(x)$ when $x = 3$ and $dx = 0.12$. Express your answer as a decimal.

$$dh = -2\cos(3-3)(0.12) = -2(1)(0.12) = -0.24$$

c. Explain what the number in part (b) indicates about the function $h(x)$.

Using the tangent line to $h(x)$ at $x=3$, the differential estimates that if x changes by 0.12, we expect $h(x)$ to decrease by 0.24.

3. [8 points] Let $f(x) = (4 - x^2)^2$.

a. Find all critical points for $f(x)$.

$$f'(x) = 2(4-x^2)'(-2x) = -4x(4-x^2) = -4x(2-x)(2+x)$$

crit. pts are $x = 0, 2, -2$

b. Determine the absolute maximum and absolute minimum of $f(x)$ on the interval $[0,3]$ or state that none exist. You must show your work to receive full credit. See the answer-blank below.

x	y
0	$4^2 = 16$
2	$0^2 = 0$
3	$(4-9)^2 = 25$

maximum value of $f(x)$: ____25____

minimum value of $f(x)$: ____0____