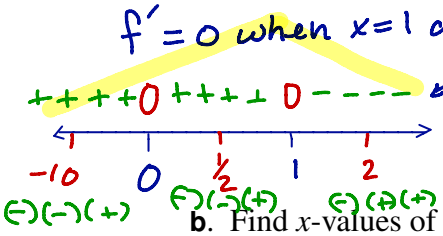
Name: Solutions _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[8 points]** Answer the questions below about the function $f(x) = 4x^3 - 3x^4$. Observe that $f'(x) = -12(x-1)x^2$ and $f''(x) = 12x(2-3x)$.

   a. Find intervals where f is increasing or decreasing.

   $f' = 0$ when $x = 1$ and $x = 0$

   Answer:
   f is ↑ on $(-\infty, 1)$ and f ↓ on $(1, \infty)$

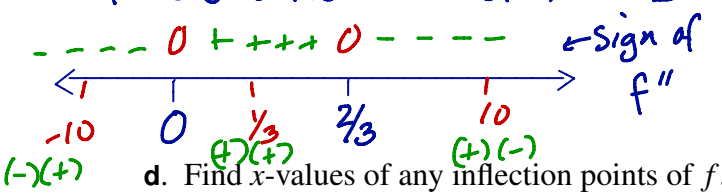
   b. Find x-values of any local minima and local maxima of f or state that none exist

   f has a local max at $x = 1$

   f has no local min

   c. Find intervals where f is concave up and concave down.

   $f'' = 0$ when $x = 0$ or $x = 2/3$

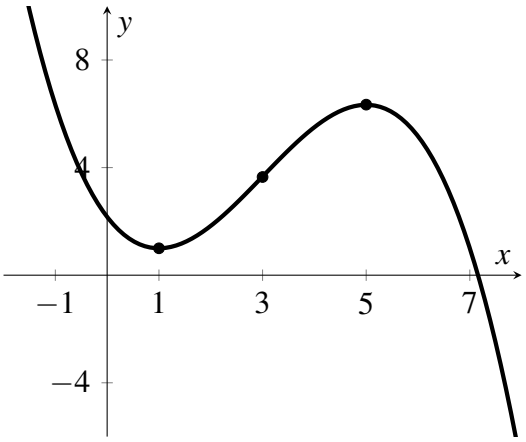
   

   Answer:
   f is c cup on $(0, 2/3)$
   f is c c down on $(-\infty, 0) \cup (\frac{2}{3}, \infty)$

   d. Find x-values of any inflection points of f.

   f has inflection points at $x = 0$ and $x = 2/3$.

2. **[6 points]** Based on the graph of the function $g(x)$ (below) to determine whether each value below is **positive**, **negative**, **zero**, or **undefined**.

   a. $g'(1) = 0$

   b. $g''(1) > 0$

   c. $g'(3) > 0$

   d. $g''(3) = 0$

   e. $g'(5) = 0$

   f. $g''(5) < 0$

3. [8 points] Evaluate the limits below. Use algebra to justify your answer.

a. $\lim_{x\to-\infty}\dfrac{x^4+1}{x^2-2x^3}$

$$\lim_{x\to-\infty}\frac{(x^4+1)\frac{1}{x^3}}{(x^2-2x^3)\frac{1}{x^3}} = \lim_{x\to-\infty}\frac{x+\frac{1}{x^3}}{\frac{1}{x}-2} = +\infty$$

($x\to-\infty$; $x+\frac{1}{x^3}\to-\infty+0$; $\frac{1}{x}-2\to0-2$)

b. $\lim_{x\to\infty}\dfrac{\sqrt{2x^6+x}}{1+x^3}$

$$\lim_{x\to\infty}\frac{\sqrt{2x^6+x}}{1+x^3}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty}\frac{\sqrt{2+\frac{1}{x^5}}}{\frac{1}{x^3}+1} = \sqrt{2}$$

($\sqrt{2+\frac{1}{x^5}}\to\sqrt{2+0}$; $\frac{1}{x^3}+1\to0+1$)

4. [3 points] Find any horizontal asymptotes of the graph $H(x) = 5 + \frac{x}{2x+1}$. Show your work.

$$\lim_{x\to\infty}\left(5+\frac{x}{2x+1}\right) = 5+\lim_{x\to\infty}\frac{1}{2+\frac{1}{x}} = 5+\frac{1}{2} = \frac{11}{2}$$

$$= 5.5$$

Asymptote: $y = 5.5$