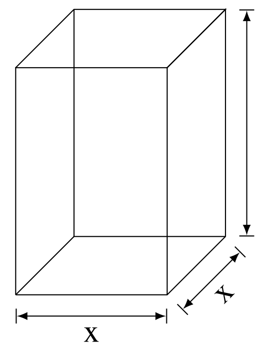
Name: Solutions ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[9 points]**

An **open-topped** box with a square base will be constructed from material that costs \$10 per square meter for the base of the box and \$1 per square meter for the sides of the box. Determine the dimensions of the box of the least expensive box that has a volume of 40 cubic meters.



a. What is a formula for the cost, C, of the box using x and y as labeled in the picture?

$$C = 10x^2 + 4xy$$

b. Write C as a function of **one** variable. You must show your work to receive any credit here.

$V = x^2 y = 40$

$y = 40x^{-2}$

$C(x) = 10x^2 + 4x(40x^{-2})$

$= 10x^2 + 160x^{-1}$

c. What is a reasonable domain for the function above?

$x > 0$ or $(0, \infty)$

d. In one approach, the function for cost could be $c(x) = 10x + 160x^{-1}$. Use this function to answer the question. **You must justify your answer to earn full credit.**

$C'(x) = 20x - 160x^{-2} = 0$

$20x = \frac{160}{x^2}$

$x^3 = \frac{160}{20} = 8$

$x = 2$

--- 0 +++ ← Sign of C'

1 2 10

So $C(x)$ has a min. at $x = 2$.

Answer: Dimensions of min. cost:

$x = 2$ m, $y = 10$ m

2. [10 points] Evaluate the limits below. If you use L'Hopital's Rule, demonstrate this by identifying the form of the limit and with an h over the equal sign.

a. $\lim_{\theta\to 0} \frac{2\theta}{\sin(\theta)} \stackrel{h}{=} \lim_{\theta\to 0} \frac{2}{\cos(\theta)} = \frac{2}{1} = 2$

form $\frac{0}{0}$

b. $\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1}} \stackrel{h}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-x^{-2}} = \lim_{x\to 0^+} \frac{-x^2}{x} = \lim_{x\to 0^+} -x = 0$

form $0\cdot\infty$ form $\frac{0}{0}$

c. $\lim_{x\to 0} \frac{x^3}{1+\cos(x)} = \frac{0}{1+1} = \frac{0}{2} = 0$

3. [6 points] Evaluate the integrals below and **check** that your answer is correct.

a. $\int (5+\sin(x))\,dx = 5x - \cos(x) + C$

check $y = 5x - \cos(x) + C$

$y' = 5 + \sin(x)$ ✓

b. $\int 4x^{1/3} - \sec^2(x)\,dx = 4\left(\frac{3}{4}\right)x^{4/3} - \tan(x) + C$

Check: $y = 3x^{4/3} - \tan(x) + C$

$y' = 3\left(\frac{4}{3}\right)x^{1/3} - \sec^2(x)$ ✓