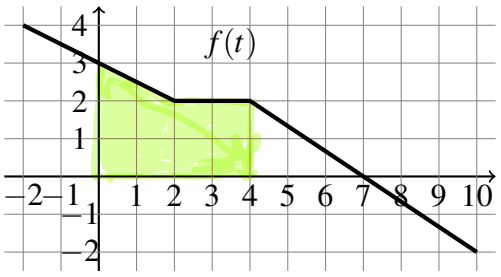
Name: Solutions _____ / 25

There are 25 points possible on this quiz. *You should be able to complete it without using your notes or textbook or a calculator — this is practice for your exams!* If you needed to look something up, you should to me about questions you might have. **Show all work for full credit** and use some words or sentences to help communicate your answers.

1. [4 points] Define $G(x) = \int_0^x f(t)\,dt$ where the graph of $f(t)$ is drawn below.



a. Determine $G(4)$.

$G(4)$ = shaded area = 9

b. Does $G(x)$ have a maximum on the interval $[0, 10]$? Explain your answer.

Yes, at $x=7$ since the derivative $f(x)$ goes from + to −

2. [6 points] Evaluate each definite integral using the Fundamental Theorem of Calculus Part 2.

a. $\int_1^9 \frac{6}{\sqrt{x}}\,dx = \int_1^9 6x^{-1/2}\,dx = \frac{6x^{1/2}}{1/2}\Big|_1^9 = 12\sqrt{x}\Big|_1^9$

$= 12\sqrt{9} - 12\sqrt{1} = 12(3-1) = \boxed{24}$

b. $\int_0^{\pi/3} (12 - 2\sin(x))\,dx = 12x + 2\cos(x)\Big|_0^{\pi/3}$

$= 12(\pi/3) + 2\cos(\pi/3) - 12(0) - 2\cos(0)$

$= 4\pi + 2(1/2) - 0 - 2$

$= \boxed{4\pi - 1}$

3. [4 points] Evaluate $\int_0^{\pi/4} (sec(\theta))^2 \tan(\theta)\,d\theta$. Show your work.

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$x = 0 \Rightarrow u = 0$

$x = \pi/4 \Rightarrow u = \tan(\pi/4) = 1$

So $\int_0^{\pi/4} \sec^2\theta \tan\theta\,d\theta$

$= \int_0^1 u\,du = \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$

Method #2:
Alternately: Let $u = \sec\theta$. Then $du = \sec\theta\tan\theta\,d\theta$
And $\theta = 0 \Rightarrow u = 1$ and $\theta = \pi/4 \Rightarrow u = \sqrt{2}$

So $\int_0^{\pi/4} \sec^2\theta\tan\theta\,d\theta =$

$\int_1^{\sqrt{2}} u\,du = \frac{u^2}{2}\Big|_1^{\sqrt{2}} = 1 - \frac{1}{2}$

$= 1/2.$

Method 3 for #3: $\int_0^{\pi/4} \sec^2\theta \tan\theta \, d\theta = \int_0^{\pi/4} \frac{1}{(\cos\theta)^2} \cdot \frac{\sin\theta}{\cos\theta} \, d\theta = \int_0^{\pi/4} \frac{\sin\theta}{(\cos\theta)^3} \, d\theta$

Let $u = \cos\theta$ and go from there!

4. [6 points] Use the Fundamental Theorem of Calculus (Part 1) to find each derivative.

a. $\frac{d}{dx}\left(\int_6^x t^5 - \frac{2}{t}\, dt\right) = \boxed{x^5 - \frac{2}{x}}$

b. $\frac{d}{dx}\left(\int_{\cos(x)}^4 \sqrt{1-t^2}\, dt\right) = -\frac{d}{dx}\int_4^{\cos x} \sqrt{1-t^2}\, dt$

$= -\sqrt{1-(\cos(x))^2}\,(-\sin(x)) = \sin(x)\sqrt{1-\cos^2 x}$

5. [5 points] A ball is thrown upward from an initial height of 4 ft at an initial speed of 10 ft/s. The acceleration due to gravity is 32 ft/s^2. (Just to be clear, we are assuming $a(t) = -32$ is the equation modeling the acceleration of the ball.)

a. Solve for $v(t)$, the velocity of the ball t seconds after it is thrown into the air. (Use calculus techniques.)

$a(t) = -32$.

$v(t) = \int a(t)\, dt = -32t + C$

but $v(0) = 10 \Rightarrow \boxed{v(t) = -32t + 10}$

b. Solve for $h(t)$, the height of the ball t seconds after it is thrown into the air. (Use calculus techniques.)

$h(t) = \int v(t)\, dt = \int -32t + 10\, dt = -32t^2 + 10t + C$

but $h(0) = 4 \Rightarrow \boxed{h(t) = -32t^2 + 10t + 4}$

c. At what time is the ball the highest? Show your work, and answer the question with a sentence.

need $v(t) = 0 \Rightarrow -32t + 10 = 0 \Rightarrow t = 10/32 = \frac{5}{16}$

The ball is highest at $5/16$ s after it was thrown.