Name: Key ________ / 25

Please circle your instructor's name: Leah Berman Jill Faudree James Gossell

There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. **For full credit, show all work in a way someone else can follow it.**

1. (12 points) Compute the derivatives of the following functions:

(a) $f(x) = 3\sec(x) - \sin(x) + \tan(\pi/4)$

$$f'(x) = 3\sec x \tan x - \cos x + 0$$

(b) $g(x) = \left(x^4 - 6x + x^{-\frac{1}{3}}\right)^5$

$$g'(x) = 5\left(x^4 - 6x + x^{-\frac{1}{3}}\right)^4 \cdot \left(4x^3 - 6 - \frac{1}{3}x^{-\frac{4}{3}}\right)$$

(c) $y = \tan\left(\dfrac{x^4 - 7}{x}\right)$

$$\frac{dy}{dx} = \sec^2\left(\frac{x^4-7}{x}\right) \cdot \frac{4x^3 \cdot x - (x^4 - 7)}{x^2}$$

(d) $r(\theta) = \theta^3 \cot(2\theta)$

$$r'(\theta) = 3\theta^2 \cot(2\theta) + \theta^3\left(-\csc(2\theta)\cot(2\theta) \cdot 2\right)$$

2. (9 points) A giant pendulum is pulled back and released so that it begins swinging back and forth. Its horizontal position is given by

$$x(t) = a\cos\left(\frac{t}{2}\right)$$

where a is a constant representing the initial horizontal position, t measures time in seconds, and x measures the position to the right of equilibrium in feet. (See the diagram below.)

(a) Find $\frac{dx}{dt}$, the derivative of the horizontal position function.

$$\frac{dx}{dt} = -a\sin\left(\frac{t}{2}\right)\cdot\frac{1}{2}$$

(b) Using your answer in part (a), find the **initial horizontal velocity** of the pendulum. Interpret your answer and explain if this makes sense in the context of the problem.

$$\left.\frac{dx}{dt}\right|_{t=0} = -a\sin(0)\cdot\frac{1}{2} = 0 \text{ ft/sec} \rightarrow$$ Yes, this make sense since the pendulum starts at rest.

(c) After π seconds, the pendulum is moving to the left at a rate of 14 feet per second. Using this information, solve for the **initial position** a.

$$\left.\frac{dx}{dt}\right|_{t=\pi} = -a\sin\left(\frac{\pi}{2}\right)\cdot\frac{1}{2} = -\frac{a}{2} = -14 \Rightarrow a = 28$$

The pendulum starts 28 feet to the right of equilibrium.

3. (4 points) Suppose $f(x)$ and $g(x)$ are differentiable functions, and $h(x) = f\left(\sqrt{g(x)}\right)$. Given that $f(3) = 3$, $f'(3) = 6$, $g(3) = 9$, and $g'(3) = 12$, **find $\mathbf{h'(3)}$** and show your work.

$$h'(x) = f'\left(\sqrt{g(x)}\right)\cdot\frac{1}{2\sqrt{g(x)}}\cdot g'(x)$$

$$h'(3) = f'(\sqrt{9})\cdot\frac{1}{2\sqrt{9}}\cdot 12$$

$$= f'(3)\cdot\frac{12}{6} = 6\cdot 2 = \boxed{12}$$