Name: Solutions _____ / 25

Please circle your instructor's name: Leah Berman Jill Faudree James Gossell

There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. **For full credit, show all work in a way someone else can follow it.**

1. (12 points) Compute the derivatives of the following functions:

(a) $f(x) = \dfrac{x + \arcsin(3x)}{5}$ $= \frac{1}{5}\left(x + \arcsin(3x)\right)$

$$f'(x) = \frac{1}{5}\left[1 + \frac{1}{\sqrt{1-(3x)^2}} \cdot 3\right] = \frac{1}{5}\left(1 + \frac{3}{\sqrt{1-9x^2}}\right)$$

(b) $g(x) = 5^{2x} - 3x^2$

$$g'(x) = \ln(5) \cdot 5^{2x} \cdot 2 - 6x = 2\ln(5)\, 5^{2x} - 6x$$

(c) $y = e^{-x}\sin(x^2)$

$$y' = -e^{-x}\sin(x^2) + e^{-x}\left(\cos(x^2)\right)(2x) = -e^{-x}\sin(x^2) + 2x e^{-x}\cos(x^2)$$

$$= e^{-x}\left(2x\cos(x^2) - \sin(x^2)\right)$$

(d) $s(t) = \ln\left(\sqrt{t^2+t}\right) = \frac{1}{2}\ln(t^2+t)$

$$s'(t) = \frac{1}{2}\left(\frac{1}{t^2+t}\right)(2t+1) = \frac{2t+1}{2(t^2+t)}$$ OR

$$s'(t) = \frac{1}{(t^2+t)^{1/2}} \cdot \frac{1}{2}(t^2+t)^{-1/2}(2t+1)$$

2. (6 points) Use implicit differentiation to find $\frac{dy}{dx}$ for $\boxed{x+\cos(xy)=y^2+x^2}$.

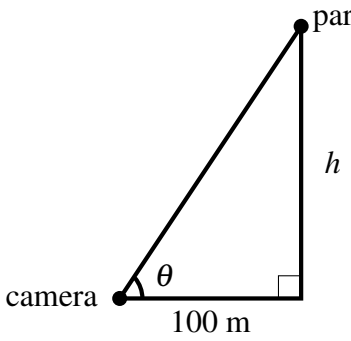
$1-\sin(xy)\left[1\cdot y+x\cdot\frac{dy}{dx}\right]=2y\frac{dy}{dx}+2x$

$1-y\sin(xy)-x\sin(xy)\frac{dy}{dx}=2y\frac{dy}{dx}+2x$

$(-x\sin(xy)-2y)\frac{dy}{dx}=2x-1+y\sin(xy)$

$\frac{dy}{dx}=\frac{2x-1+y\sin(xy)}{-x\sin(xy)-2y}=\frac{1-2x-y\sin(xy)}{x\sin(xy)+2y}$

3. (7 points) A camera at ground level is 100 meters from the landing site of a parachutist who is landing vertically. Let h be the height of the parachutist above the ground and let θ be the angle of elevation formed between the camera lens and the ground. (See figure.)



(a) Find an equation relating h and θ and **solve it for** θ.

$\tan(\theta)=\frac{h}{100}$

$\theta=\arctan\left(\frac{h}{100}\right)$

(b) Find $\frac{d\theta}{dh}$.

$\frac{d\theta}{dt}=\frac{1}{1+\left(\frac{h}{100}\right)^2}\cdot\frac{1}{100}=\frac{1}{100+\frac{h^2}{100}}=\frac{100}{(100)^2+h^2}$

(c) What are the **units** of $\frac{d\theta}{dh}$? $\frac{\text{radians}}{\text{meters}}$