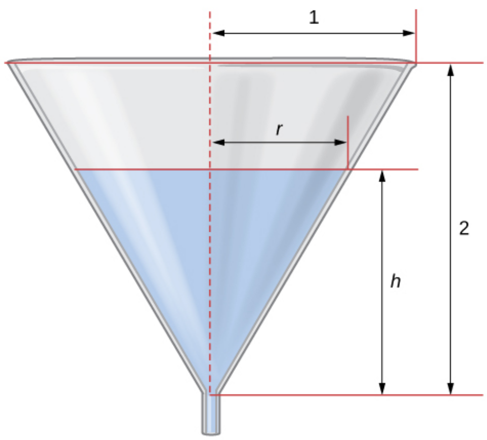
**Name:** Solutions ______ / 25

**Please circle your instructor's name:** Leah Berman Jill Faudree James Gossell

There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. **For full credit, show all work in a way someone else can follow it.**

1. (10 points) Water is draining from the bottom of a cone-shaped funnel at a rate of 0.1 cubic feet per second. The height of the funnel is 2 feet and the radius at the top of the funnel is 1 foot.




Note that the formula for the volume of water in the cone is given by $V = \frac{1}{3}\pi r^2 h$.

Note that you can use similar triangles to find a relationship between $r$ and $h$.

$$\frac{r}{h} = \frac{1}{2} \Rightarrow r = \frac{1}{2}h$$

a) **Find the rate at which the height of the water in the funnel is changing** when the height of the water ($h$) is 1 foot.

$$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h \Rightarrow V = \frac{1}{12}\pi h^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{1}{12}\pi h^3\right]$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} \xrightarrow{\text{Plugging in } -0.1 \text{ for } \frac{dV}{dt} \text{ and } 1 \text{ for } h} -0.1 = \frac{1}{4}\pi(1)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-0.4}{\pi} \text{ feet per second}$$

b) Using complete sentances, **explain what your answer in part (a) means** in the context of this problem. Include units in your explamnation.

When the height of the water is 1 foot, the height of the water is decreasing at a rate of $\frac{0.4}{\pi}$ feet per second.

2. (7 points) Complete the following steps to approximate $\sqrt[3]{30}$ without a calculator:

(a) Find the linear approximation $L(x)$ to $f(x) = \sqrt[3]{x}$ at $a = 27$.

$f(27) = \sqrt[3]{27} = 3$

$f'(x) = \frac{1}{3} x^{-2/3}$

$f'(27) = \frac{1}{3\sqrt[3]{27^2}} = \frac{1}{3 \cdot 9} = \frac{1}{27}$

$\boxed{L(x) = 3 + \frac{1}{27}(x - 27)}$

(b) Use $L(x)$ to approximate $\sqrt[3]{30}$. Write your answer as a fraction.

$L(30) = 3 + \frac{1}{27}(30 - 27) = 3 + \frac{3}{27} = 3 + \frac{1}{9} = \boxed{\frac{28}{9}}$

$\sqrt[3]{30} \approx \frac{28}{9}$

3. (8 points) Find the absolute maxima and minima for the function $f(x) = x(x-4)^3$ over the interval $[0, 5]$. Show your work, including relevant computations.

$f'(x) = (x-4)^3 + x \cdot 3(x-4)^2 = (x-4)^2(x - 4 + 3x) = 4(x-4)^2(x-1)$

Critical values: $f'(x)$ is never undefined, $f'(x) = 0$ at $\underline{x = 1}$ and $\underline{x = 4}$

Testing critical values and endpoints:

| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 1 | -27 |
| 4 | 0 |
| 5 | 5 |

**maximum value of $f(x)$:** 5 (at $x = 5$)

**minimum value of $f(x)$:** -27 (at $x = 1$)