Name: ______________________ ______ / 25

Please circle your instructor's name: Leah Berman Jill Faudree James Gossell

There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. **For full credit, show all work in a way someone else can follow it.**

1. (12 points) Answer the questions below about the function $f(x) = x^3(x+2)$. After simplification,

$$f'(x) = 2x^2(2x+3), \qquad \text{and } f''(x) = 12x(x+1).$$

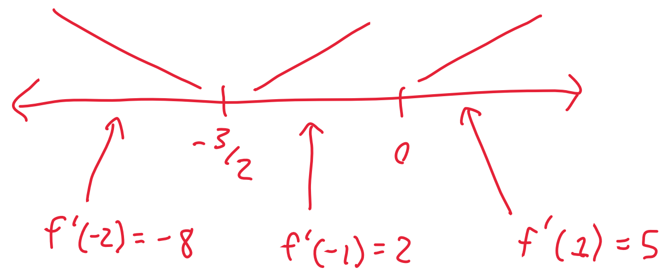
You must show your work and justify your conclusion with a few words or a computation. Make sure someone else can follow your work.

(a) Determine the intervals where f is **increasing** and where f is **decreasing**. Show your work.

$f'(x) = 0 \Rightarrow 2x^2(2x+3) = 0$

$2x^2 = 0$ $\quad$ $2x+3=0$

$x = 0$ $\quad$ $x = -3/2$



Increasing: $(-\frac{3}{2}, 0) \cup (0, \infty)$ Decreasing: $(-\infty, -\frac{3}{2})$

(Use interval notation. If none write "none".)

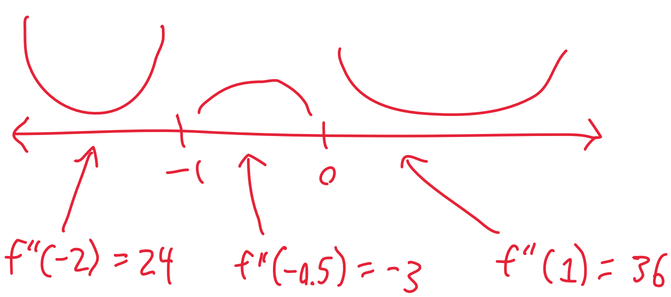
(b) Fill in the blanks: $f(x)$ has a local maximum at $x =$ none and a local minimum at $x =$ $-3/2$. (If none, write "none".)

(c) Find all intervals where f is **concave up** and where f is **concave down**. Show your work.

$f''(x) = 0 \Rightarrow 12x(x+1) = 0$

$12x = 0$ $\quad$ $x+1 = 0$

$x = 0$ $\quad$ $x = -1$



Concave up: $(-\infty, -1) \cup (0, \infty)$ Concave down: $(-1, 0)$

(Use interval notation. If none write "none".)

(d) Fill in the blanks: $f(x)$ has (an) inflection point(s) at $x =$ $0, -1$. (If none, write "none".)

2. (6 points)

(a) Determine $\lim\limits_{x\to\infty} \dfrac{\sqrt{3x^2+1}}{4x-2}$. Show some work.

$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{4x-2}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{\sqrt{\frac{3x^2+1}{x^2}}}{4-\frac{2}{x}} = \lim_{x\to\infty}\frac{\sqrt{3+\cancelto{0}{\frac{1}{x^2}}}}{4-\cancelto{0}{\frac{2}{x}}} = \frac{\sqrt{3}}{4}$$

(b) Fill in the empty boxes to make a true sentence.

The function $g(x) = \dfrac{\sqrt{3x^2+1}}{4x-2}$ has a horizontal asymptote whose equation is $\boxed{y = \frac{\sqrt{3}}{4}}$ because

$$\lim_{\boxed{x\to\infty}} g(x) = \boxed{\sqrt{3}/4}.$$

3. (7 points) Sketch a graph of a function $h(x)$ with the following properties:

- The domain of $h(x)$ is $(-\infty,3)\cup(3,\infty)$.
- $h(0) = 1$
- $h(1) = 2$
- $\lim\limits_{x\to-\infty} h(x) = 0$
- $\lim\limits_{x\to\infty} h(x) = -2$
- $\lim\limits_{x\to 3^-} h(x) = -\infty$
- $\lim\limits_{x\to 3^+} h(x) = \infty$
- $h'(x) > 0$ when $x < 1$.
- $h'(x) < 0$ when $1 < x < 3$ or $x > 3$.
- $h''(x) > 0$ when $x < 0$ or $x > 3$.
- $h''(x) < 0$ when $0 < x < 3$

- **Label** on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- **Draw** any horizontal and vertical asymptotes with dashed lines and **label** them with their equation.
- **Mark** any important x-values and y-values on the x- and y-axes.