There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. For full credit, show all work in a way someone else can follow it.

1. (12 points) Answer the questions below about the function $f(x) = x^3(x+2)$. After simplification,

 $f'(x) = 2x^2(2x+3)$, and f''(x) = 12x(x+1).

You must show your work and justify your conclusion with a few words or a computation. Make sure someone else can follow your work.

(a) Determine the intervals where f is **increasing** and where f is **decreasing**. Show your work.

Increasing:	_Decreasing:
(Use interval notation. If none write "none".)	

- (b) Fill in the blanks: f(x) has a local maximum at x = _____ and a local minimum at x = _____ (If none, write "none".)
- (c) Find all intervals where f is **concave up** and where f is **concave down**. Show your work.

Concave up: _____Concave down:_____

(Use interval notation. If none write "none".)

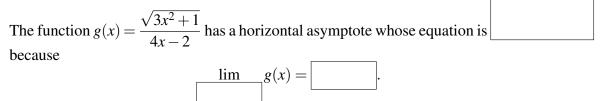
(d) Fill in the blanks: f(x) has (an) inflection point(s) at x = _____. (If none, write "none".)

October 31, 2024 🕮

2. (6 points)

(a) Determine
$$\lim_{x \to \infty} \frac{\sqrt{3x^2 + 1}}{4x - 2}$$
. Show some work.

(b) Fill in the empty boxes to make a true sentence.



- 3. (7 points) Sketch a graph of a function h(x) with the following properties:
 - The domain of h(x) is $(-\infty, 3) \cup (3, \infty)$. $\lim_{x \to 3^+} h(x) = \infty$
 - h(0) = 1
 - h(1) = 2

- h'(x) > 0 when x < 1.
- h'(x) < 0 when 1 < x < 3 or x > 3.
- $\lim_{x \to -\infty} h(x) = 0$ • $\lim h(x) = -2$ • h''(x) > 0 when x < 0 or x > 3. $x \rightarrow \infty$
- $\lim h(x) = -\infty$ • h''(x) < 0 when 0 < x < 3 $x \rightarrow 3^{-}$
- Label on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- Draw any horizontal and vertical asymptotes with dashed lines and label them with their equation.
- Mark any important x-values and y-values on the x- and y-axes.

