Name: _____ _____ **/ 25**

Please circle your instructor's name: Leah Berman Jill Faudree James Gossell

There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. **For full credit, show all work in a way someone else can follow it.**

1. (12 points) Answer the questions below about the function $f(x) = x^3(x+2)$. After simplification,
$$f'(x) = 2x^2(2x+3), \quad \text{and } f''(x) = 12x(x+1).$$
You must show your work and justify your conclusion with a few words or a computation. Make sure someone else can follow your work.

   (a) Determine the intervals where f is **increasing** and where f is **decreasing**. Show your work.

   Increasing: _____ Decreasing: _____
   (Use interval notation. If none write "none".)

   (b) Fill in the blanks: $f(x)$ has a local maximum at $x =$ _____ and a local minimum at $x =$ _____. (If none, write "none".)

   (c) Find all intervals where f is **concave up** and where f is **concave down**. Show your work.

   Concave up: _____ Concave down: _____
   (Use interval notation. If none write "none".)

   (d) Fill in the blanks: $f(x)$ has (an) inflection point(s) at $x =$ _____. (If none, write "none".)

2. (6 points)

   (a) Determine $\lim_{x\to\infty} \dfrac{\sqrt{3x^2+1}}{4x-2}$. Show some work.

   (b) Fill in the empty boxes to make a true sentence.

   The function $g(x) = \dfrac{\sqrt{3x^2+1}}{4x-2}$ has a horizontal asymptote whose equation is $\square$ because

   $$\lim_{\square} g(x) = \square.$$

3. (7 points) Sketch a graph of a function $h(x)$ with the following properties:

   - The domain of $h(x)$ is $(-\infty, 3) \cup (3, \infty)$.
   - $h(0) = 1$
   - $h(1) = 2$
   - $\lim_{x\to-\infty} h(x) = 0$
   - $\lim_{x\to\infty} h(x) = -2$
   - $\lim_{x\to3^-} h(x) = -\infty$
   - $\lim_{x\to3^+} h(x) = \infty$
   - $h'(x) > 0$ when $x < 1$.
   - $h'(x) < 0$ when $1 < x < 3$ or $x > 3$.
   - $h''(x) > 0$ when $x < 0$ or $x > 3$.
   - $h''(x) < 0$ when $0 < x < 3$

   - **Label** on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
   - **Draw** any horizontal and vertical asymptotes with dashed lines and **label** them with their equation.
   - **Mark** any important x-values and y-values on the x- and y-axes.