Name: Solutions ________ / 25

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There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. **For full credit, show all work in a way someone else can follow it.**

constraint

1. (9 points) Find the radius, r, and height, h, of the open-topped cylinder with $\boxed{\text{volume } 8\pi}$ that has the least amount of surface area. The formulas for the volume, V, and surface area, S, are given below.

$$V = \pi r^2 h, \quad S = \pi r^2 + 2\pi r h$$

- goal: minimize S
- Write S as a function of one variable.

Use constraint: $8\pi = \pi r^2 h$.

Solve for h: $h = 8r^{-2}$

Plug into S: $S(r) = \pi r^2 + 2\pi r(8r^{-2}) = \pi r^2 + 16\pi r^{-1}$

Domain: $(0, \infty)$

- Do calculus

$S'(r) = 2\pi r - 16\pi r^{-2} = 0$

$2\pi r = \dfrac{16\pi}{r^2}$,

$r^3 = 8$, $r = 2$ ← crit #

Check it's a min:

$$\begin{array}{cccccc} (& - - - - & 0 & + + + + & \to & \leftarrow \text{sign of } S' \\ 0 & 1 & 2 & 10 & & \end{array}$$

$S'(1) = 2\pi - 16\pi < 0$

$S'(10) = 20\pi - \frac{16\pi}{100} > 0$

Since $r=2$ is the only extrema and it's a minimum, it's an absolute minimum.

Answer: The surface area is minimized when $r=2$ and $h=2$.

2. (8 points) Use L'Hôpital's Rule to evaluate the limits below. Indicate your use of L'Hôpital's Rule with an h above the equal sign.

(a) $\lim_{x\to 0} \frac{e^{2x}-2x-1}{5x^2} \overset{h}{=} \lim_{h\to 0} \frac{2e^{2x}-2}{10x} \overset{h}{=} \lim_{h\to 0} \frac{4e^{2x}}{10} = \frac{4}{10} = \frac{2}{5}$

form $\frac{0}{0}$ form $\frac{0}{0}$

(b) $\lim_{x\to\infty} \sqrt{x}\, e^{-x} = \lim_{x\to\infty} \frac{x^{1/2}}{e^x} \overset{h}{=} \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{e^x} = \lim_{x\to\infty} \frac{1}{2\sqrt{x}\,e^x} = 0$

form $\infty \cdot 0$ form $\frac{\infty}{\infty}$

3. (8 points) Evaluate the integrals below.

(a) $\int (3x^5 + \sin(x) + e^x + \pi^2)\,dx = \frac{3}{6}x^6 - \cos(x) + e^x + \pi^2 x + C$

(b) $\int \frac{3x + x^{1/3}}{x}\,dx = \int \frac{3x}{x} + \frac{x^{1/3}}{x}\,dx = \int (3 + x^{-2/3})\,dx$

$= 3x + \frac{x^{1/3}}{\frac{1}{3}} + C = 3x + 3x^{1/3} + C$