

Name: Solutions

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There are 16 questions worth 40 points on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit. Give exact numerical answers such as  $\sqrt{7}$  or  $\frac{5}{\pi}$ .

**Algebra**

1. [2 points] Simplify each expression below.

a. Write the expression  $\frac{(xyz)^3}{x^4y^{-2}z}$  in the form  $x^a y^b z^c$ . That is, write the expression with all terms in the numerator.

$$\frac{x^3 y^3 z^3}{x^4 y^{-2} z^1} = x^{3-4} y^{3-(-2)} z^{3-1} = x^{-1} y^5 z^2$$


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$$x^{-1} y^5 z^2$$

b. Cancel any common factors in both the numerator and denominator for the expression

$$\frac{2xy^2 + 4y^3}{3x^2 + 6xy}$$


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$$= \frac{2y^2(x+2y)}{3x(x+2y)}$$


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$$\frac{2y^2}{3x}$$

2. [2 points] Solve the following equations for x (giving exact answers).

a.  $5^x + 1 = 12$ .

$$5^x = 11$$

$$\ln(5^x) = \ln(11)$$

$$x \ln(5) = \ln(11)$$

$$x = \frac{\ln(11)}{\ln(5)}$$

or  $x = \log_5 11$

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$$x = \frac{\ln(11)}{\ln(5)}$$

b.  $\ln(x+3) = \frac{1}{2}$ .

$$x+3 = e^{\frac{1}{2}} \quad x = e^{\frac{1}{2}} - 3$$


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$$x = e^{\frac{1}{2}} - 3$$

3. [3 points] Solve the inequality  $x^2 < 4$  for x. Write your answer in interval notation.

$$-2 < x < 2$$


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$$(-2, 2)$$

Geometry and Trigonometry

4. [2 points] A circular field has an area of 83 square feet. Determine its radius. Include units with your answer.

$$A = \pi r^2 \quad r = \sqrt{\frac{83}{\pi}} \text{ ft}$$

$$83 = \pi r^2$$

$$r^2 = 83/\pi$$

$$r = \sqrt{\frac{83}{\pi}} \text{ ft}$$

5. [2 points] Write an equation of the line between the points  $(-3, 2)$  and  $(4, 0)$ .

$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{0-2}{4-(-3)} = \frac{-2}{7}$$

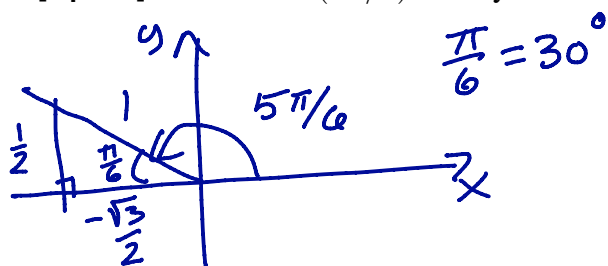
$$y = \frac{-2}{7}x + \frac{8}{7}$$

or

use  $y - y_0 = m(x - x_0)$

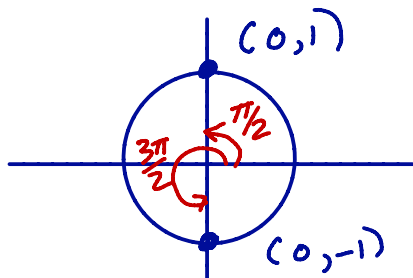
$$y = \frac{-2}{7}(x - 4)$$

6. [1 point] Evaluate  $\sin(5\pi/6)$  exactly.



$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

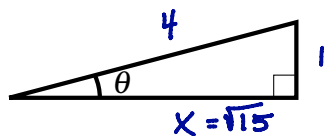
7. [2 points] Solve the equation  $\cos(x) = 0$  on the interval  $0 \leq x < 2\pi$ . Assume  $x$  is in radians.



$\cos(\theta)$  is x-value of pts on unit circle

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

8. [2 points] In the right triangle below,  $\sin(\theta) = \frac{1}{4}$ . Determine  $\tan(\theta)$ .



$$\tan \theta = \frac{\text{OPP}}{\text{adj}} = \frac{1}{\sqrt{15}}$$

$$\begin{aligned} x^2 + 1^2 &= 4^2 \\ x^2 &= 15 \end{aligned}$$

$x = \sqrt{15}$

$$\tan(\theta) = \frac{1}{\sqrt{15}}$$

Functions



9. [2 points] Determine the domain and range of  $f(x) = 3 + \sqrt{x}$ . Write your answer in interval notation.

domain  $\rightarrow \sqrt{x}$  doesn't accept negative numbers.  
 range  $\rightarrow$  The "+3" shifts  $\sqrt{x}$  up 3.

domain:  $[0, \infty)$

range:  $[3, \infty)$

10. [2 points] For  $f(x) = x - x^2$ , find  $f(a+2)$ . Simplify your answer by multiplying out and collecting like terms.

$$f(a+2) = a+2 - (a+2)^2 = a+2 - (a^2 + 4a + 4)$$

$$= a+2 - a^2 - 4a - 4$$

$$= -a^2 - 3a - 2$$

$f(a+2) = -a^2 - 3a - 2$

11. [2 points] Use the piecewise defined function  $f(x) = \begin{cases} x+1 & x \leq 0 \\ \frac{1}{x} & x > 0 \end{cases}$ .

a. Find  $f(-2.4)$ .

$-2.4 < 0$ . So use top function  
 $f(-2.4) = -2.4 + 1 = -1.4$

$f(-2.4) = -1.4$

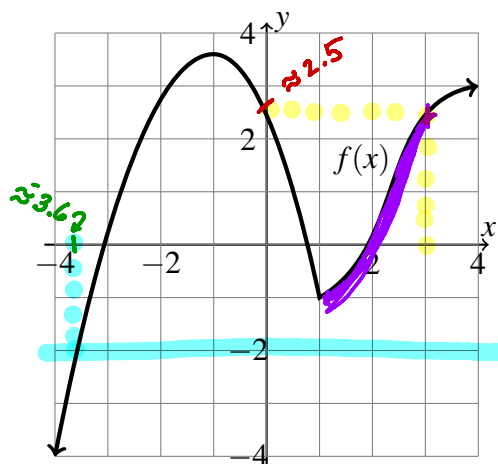
b. Determine  $x$  such that  $f(x) = 4$ .

$x+1=4$  means  $x=3$ ; not  $\leq 0$

$\frac{1}{x}=4$  means  $x=\frac{1}{4} > 0$

$x = \frac{1}{4}$

12. [3 points] Use the graph of  $f(x)$  below to answer the questions.



a. Estimate  $f(3)$ .

$y \approx 2.5$

b. Estimate an  $x$ -value such that  $f(x) = -2$ .

$x \approx -3.6$

On the interval from  $x=1$  to  $x=3$ ,

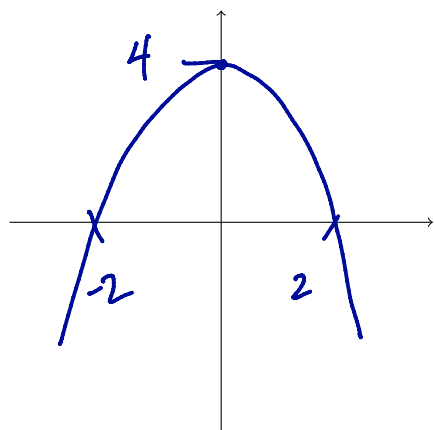
c. is  $f(x)$  increasing, decreasing, or constant?

increasing

**Graphing**

For problems 13-16, graph each function on the axes provided. Draw any asymptotes with dashed lines. Fill in the blanks identifying any  $x$ - or  $y$ -intercepts and the **equations** of any asymptotes. Write **none** if no intercepts or asymptotes exist.

13. [4 points]  $f(x) = 4 - x^2$ .   
 ← upside down parabola  
 ← shifted up 4



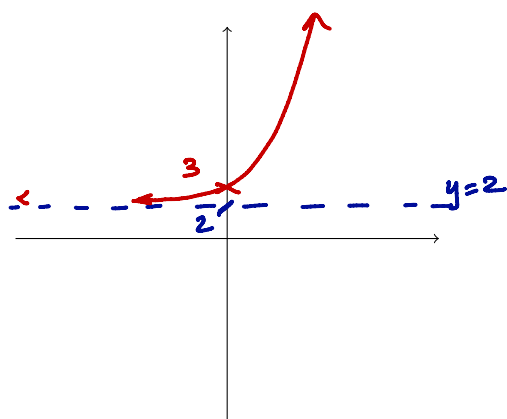
$4 - x^2 = 0$  when  $x = \pm 2$

$x$  intercepts:  $x = 2, x = -2$

$y$ -intercepts:  $y = 4$

asymptote(s): none

14. [4 points]  $f(x) = e^x + 2$  ←  $e^x$  shifted up 2



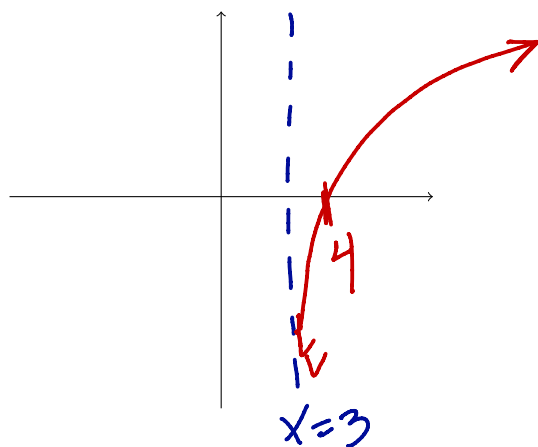
$x$  intercepts: none

$y$ -intercepts:  $y = 3$

asymptote(s):  $y = 2$  (horizontal)

15. [4 points]  $f(x) = \ln(x-3)$

$\ln(x)$  shifted right 3 units



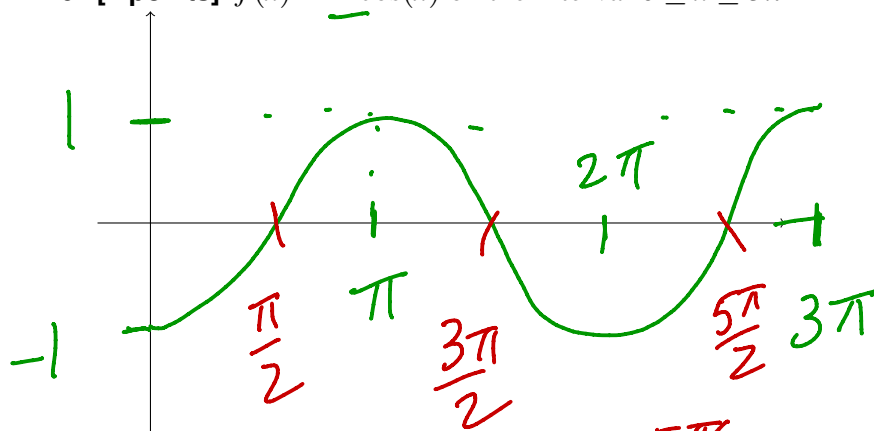
x intercepts:  $x=4$

y-intercepts: none

asymptote(s):  $x=3$  (vertical)

16. [4 points]  $f(x) = -\cos(x)$  on the interval  $0 \leq x \leq 3\pi$ .

$\cos(x)$  reflected over x-axis



x intercepts:  $x = \pi/2, 3\pi/2, 5\pi/2$

y-intercepts:  $y = -1$

asymptote(s): none