Name: Solutions _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [11 points] Let $P(2,2)$ be a point on the graph of $f(x) = \dfrac{6-x}{x}$.

a. Find the slope of the secant line passing through P and the point $Q(1, f(1))$.

$f(1) = \frac{6-1}{1} = 5$ slope $= \frac{f(1)-f(2)}{1-2} = \frac{5-2}{1-2} = \frac{3}{-1} = -3$

b. The table below lists the slope of the secant line passing through the point P and the point $Q(x, f(x))$ for several values of x.

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	2.157895	2.015075	2.001501	1.998501	1.985075	1.857143
m_{sec}	-1.57895	-1.50754	-1.50075	-1.49925	-1.49254	-1.42857

Use the information in the table to estimate the slope of the tangent line to $f(x)$ at the point $P(2,2)$.

Estimate is slope of tangent line is $-1.5 = \frac{-3}{2}$

c. Use the slope from part (c) above to write an equation of the tangent line at point $P(2,2)$.

$y = \frac{-3}{2}(x-2)+2$

Note there is no need to use slope-intercept form here. But if you want to,

$y = \frac{-3}{2}(x-2)+2$

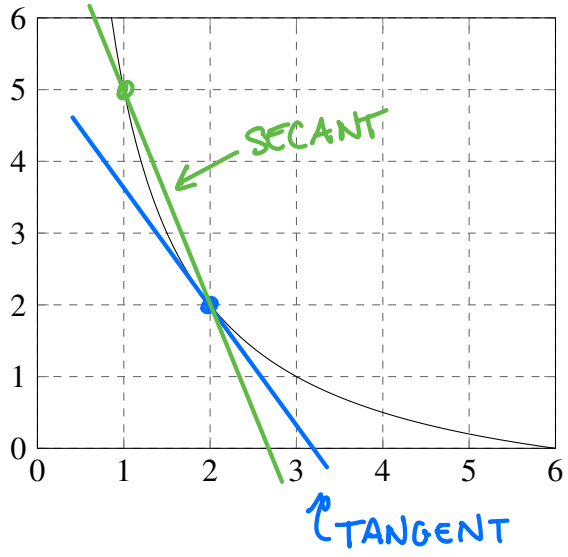
$= \frac{-3}{2}x - 3 + 2$

$= -3/2\,x - 1.$

d. Below is a sketch of the graph of $f(x) = \frac{6-x}{x}$.

(a) Sketch the **tangent line** to the graph at the point $P(2,2)$. Label it with the word TANGENT.

(b) Sketch the **secant line** passing through $P(2,2)$ and $Q(1, f(1))$. Label it with the word SECANT.



2. [8 points] The height, h, of an object is given by the expression $h(t) = 10 - \sqrt{t}$ where h is measured in meters and t is measured in seconds.

a. Compute the **average velocity** of the object over the time intervals. Include units with your answers.

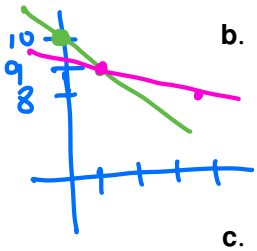
$h(0) = 10,\ h(1) = 9,\ h(4) = 8$

(i) $[0, 1]$

$$\frac{h(1) - h(0)}{1-0} = \frac{(10-\sqrt{1}) - (10-\sqrt{0})}{1-0} = \frac{10-1-10+0}{1} = -1 \text{ m/s}$$

(ii) $[1, 4]$

$$\frac{h(4) - h(1)}{4-1} = \frac{(10-\sqrt{4}) - (10-\sqrt{1})}{4-1} = \frac{8-9}{3} = \frac{-1}{3} = -1/3 \text{ m/s}$$



b. **Using the calculations you did in part (a) above,** estimate the **instantaneous velocity** of the object when $t = 1$. Include units with your answer.

We only have 2 values for average velocity, so we can estimate by averaging these: $\frac{(-1)+(-1/3)}{2} = \frac{-3/3 - 1/3}{2} = \left(-\frac{4}{3}\right)\left(\frac{1}{2}\right) = -\frac{2}{3}$ m/s

c. **What do your calculations in part (b) above** indicate about whether the object appears to be rising (or gaining height) or dropping (losing height)?

the instantaneous velocity is negative, so the object is dropping (losing height)

3. [6 points] An object is attached to a spring suspended from above. The height of the object above the ground is given by $h(t) = 4\cos(\pi t) + 6$ where h is in inches and t is in seconds.

a. Calculate the length of the spring at $t = \frac{1}{3}$ seconds and $t = \frac{2}{3}$ seconds. Include units in your final answers.

$h\left(\frac{1}{3}\right) = 4\cos(\pi/3) + 6 = 4(1/2) + 6 = 2 + 6 = 8$ inches

$h\left(\frac{2}{3}\right) = 4\cos(2\pi/3) + 6 = 4(-1/2) + 6 = -2 + 6 = 4$ inches

b. Find the **average velocity** of object over the time interval $\left[\frac{1}{3}, \frac{2}{3}\right]$. Show your work and include units in your final answer.

$$\text{average velocity} = \frac{8-4}{1/3 - 2/3} = \frac{4}{-1/3} = -12 \text{ inches/second}$$