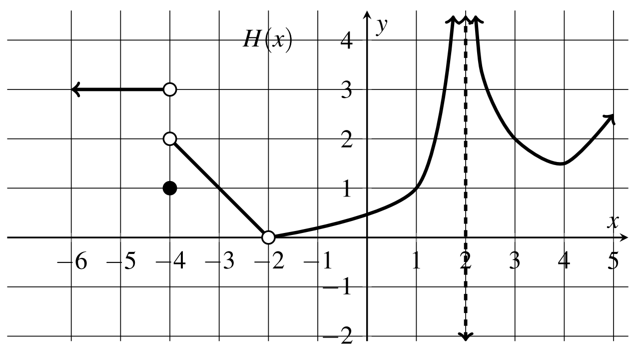
Name: Solutions ______ / 25

Please circle your instructor's name: Leah Berman Jill Faudree James Gossell

There are 25 points possible on this quiz. Any outside materials (textbook, course notes, calculator) are not allowed. **For full credit, show all work in a way someone else can follow it.**

1. (13 points) The graph of a function $H(x)$ is shown below. Use the graph of $H(x)$ to answer each question below. If the limit is infinite, indicate that with ∞ or $-\infty$. If the value does not exist or is undefined, write DNE.



(a) $\lim_{x\to -2^-} H(x) =$ 0 (b) $\lim_{x\to -2^+} H(x) =$ 0 (c) $H(-2) =$ DNE

(d) $H(-4) =$ 1 (e) $\lim_{x\to -4^+} H(x) =$ 3 (f) $\lim_{x\to -4^+} H(x) =$ 2

(g) $\lim_{x\to 2} H(x) =$ ∞

(h) Based on the information from the graph, write the domain of $H(x)$ using interval notation:
$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

(i) Observe from the graph that $\lim_{x\to 3} H(x) = 2$.

Determine $\lim_{x\to 3} \dfrac{5H(x)-1}{x^2 H(x)} =$

$$\lim_{x\to 3} \frac{5H(x)-1}{x^2 H(x)} = \frac{5(2)-1}{4\cdot 2} = \frac{9}{8}$$

(j) List all **x-values** in the set $(-\infty, \infty)$ where the function $H(x)$ is not continuous.
$x =$ $-4, -2, 2$

2. (6 points) Use algebra to evaluate the limits below. You must show your work to earn full credit **and** your work will be graded. (That is, you need to **write your mathematics** clearly and correctly. If you do not write $\lim\limits_{x\to\dots}\cdots$ where it is necessary your answer will not be completely correct.)

   (a) $\lim\limits_{x\to 3}\dfrac{x^2+x-6}{(x+3)^2} = \dfrac{3^2+3-6}{(3+3)^2} = \dfrac{12-6}{36} = \dfrac{6}{36} = \boxed{6}$

   (b) $\lim\limits_{h\to 0}\dfrac{\frac{4}{h+5}-\frac{4}{5}}{h} = \lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{4}{h+5}\left(\dfrac{5}{5}\right) - \dfrac{4}{5}\left(\dfrac{h+5}{h+5}\right)\right)$

   $= \lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{20-4(h+5)}{5(h+5)}\right) = \lim\limits_{h\to 0}\dfrac{1}{h}\dfrac{(20-4h-20)}{5(h+5)}$

   $= \lim\limits_{h\to 0}\dfrac{1}{\cancel{h}}\left(\dfrac{-4\cancel{h}}{5(h+5)}\right) = \lim\limits_{h\to 0}\dfrac{-4}{5(h+5)} = \boxed{\dfrac{-4}{25}}$

3. (6 points) Let

$$f(x) = \begin{cases} \frac{x^2+4x-5}{(x+6)(x-1)} & x<1 \\ 3\ln(x) & x\geq 1 \end{cases}.$$

Show your work clearly, using limit notation, to answer the following:

   (a) $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}\dfrac{x^2+4x-5}{(x+6)(x-1)} = \lim\limits_{x\to 1^-}\dfrac{(x+5)\cancel{(x-1)}}{(x+6)\cancel{(x-1)}}$

   $= \lim\limits_{h\to 1^-}\dfrac{x+5}{x+6} = \dfrac{1+5}{1+6} = \boxed{\dfrac{6}{7}}$

   (b) $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} 3\ln(x) = 3\ln(1) = 0$

   (c) $f(1) = 0$

   (d) Based on your answers to parts (a), (b) and (c), **check the true statement(s) below**:

   - ☐ f is continuous at $x=1$.
   - ☐ f has a removable discontinuity at $x=1$.
   - ☒ f has a jump discontinuity at $x=1$.
   - ☐ f has an infinite discontinuity at $x=1$.
   - ☐ None of the above.