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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [4 points]

a. Why is the following not a true statement? $\dfrac{x^2-6x}{x} = x-6$

We can't set $x=0$ in the left-hand side but $x=0$ is legal in the right-hand side.

b. Nevertheless, explain why the following equation is correct. $\displaystyle\lim_{x\to 0}\frac{x^2-6x}{x} = \lim_{x\to 0} x-6$

Limits of expressions that differ at a single point are the same. "Limits don't care about one point."

2. [4 points] Compute $\displaystyle\lim_{x\to 5}\frac{\frac{1}{5}-\frac{1}{x}}{5-x}$.

$$\lim_{x\to 5}\frac{\frac{1}{5}-\frac{1}{x}}{5-x} = \lim_{x\to 5}\frac{\frac{x-5}{5x}}{5-x}$$

$$= \lim_{x\to 5}\frac{-\frac{(5-x)}{5x}}{(5-x)}$$

$$= \lim_{x\to 5} -\frac{1}{5x} = \boxed{-\frac{1}{25}}.$$

3. [4 points] Compute $\displaystyle\lim_{h\to 0}\frac{(2+h)^2-4}{h}$

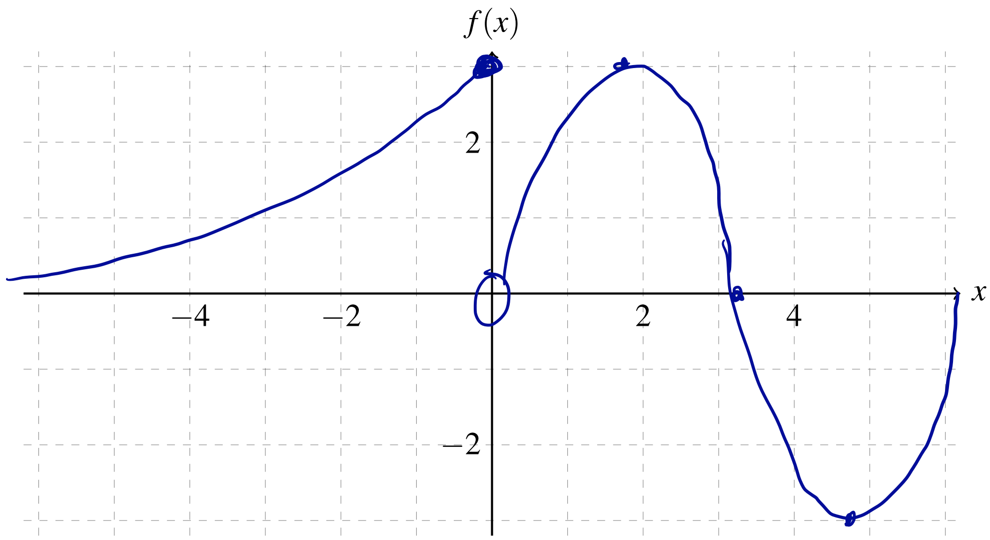
$$\lim_{h\to 0}\frac{(2+h)^2-4}{h} = \lim_{h\to 0}\frac{4+4h+h^2-4}{h}$$

$$= \lim_{h\to 0}\frac{(4+h)h}{h}$$

$$= \lim_{h\to 0} 4+h = 4.$$

4. [6 points] Consider the function $f(x) = \begin{cases} \frac{3}{1-x} & x \le 0 \\ 3\sin(x) & x > 0. \end{cases}$

a. In the diagram below, graph $f(x)$.



b. Explain why $f(x)$ isn't continuous at $x = 0$.

$\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$ so $\lim_{x \to 0} f(x)$ does not exist

5. [4 points] Use the Intermediate Value Theorem to justify the claim that there exists a number x satisfying $2^x - x - 4 = 0$.

Let $f(x) = 2^x - x - 4$.

Notice $f(0) = 2^0 - 0 - 4 = -3 < 0$ and

$f(3) = 2^3 - 3 - 4 = 1 > 0$.

Since $f(x)$ is continuous, there is an x in $[0, 3]$ where $f(x) = 0$.