Name: ______________________________ ________ / 22

There are 22 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [4 points]

a. Why is the following not a true statement? $\frac{(x-3)(x-2)}{x-2} = x-3$

We cannot set $x=2$ in the left-hand side, but we can set $x=2$ in the right-hand side.

b. Nevertheless, explain why the following equation is correct. $\lim_{x\to 0} \frac{(x-3)(x-2)}{x-2} = \lim_{x\to 0} x-3$

Limits of expressions that differ at one point are the same.
"Limits don't care about one point."

2. [4 points] Compute $\lim_{x\to 2} \frac{x^2-4}{x-2}$

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2}$$

$$= \lim_{x\to 2} x+2$$

$$= 4$$

3. [4 points] Compute $\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$.

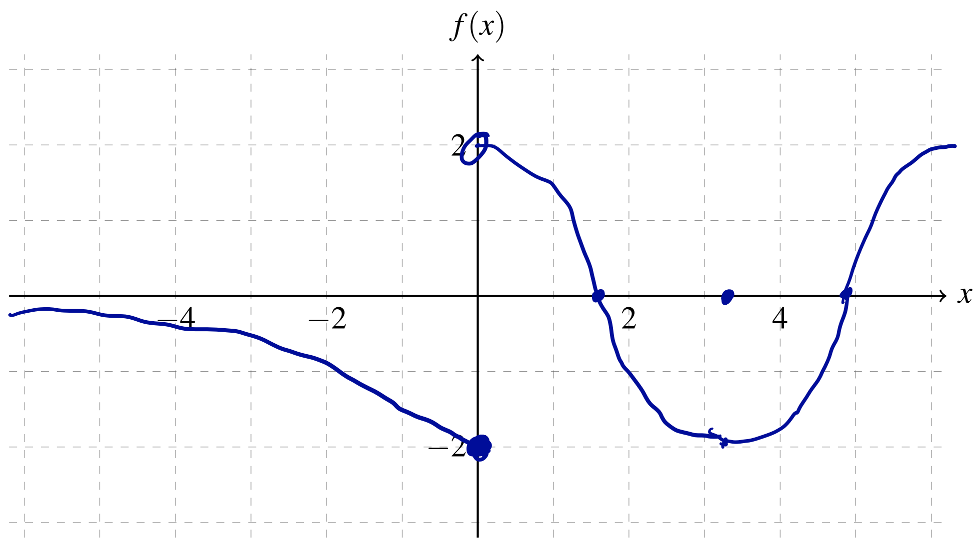
$$\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{\frac{5-(5+h)}{5(5+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-h}{5(5+h)} \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-1}{5(5+h)} = -\frac{1}{25}$$

4. [6 points] Consider the function $f(x) = \begin{cases} \frac{2}{x-1} & x \leq 0 \\ 2\cos(x) & x > 0. \end{cases}$

a. In the diagram below, graph $f(x)$.



b. Explain why $f(x)$ isn't continuous at $x = 0$.

$\lim_{x\to 0^+} f(x) \neq \lim_{x \to 0^+} f(x)$ So $\lim_{x\to 0} f(x)$ does not exist.

5. [4 points] Use the Intermediate Value Theorem to justify the claim that there exists a number x satisfying $\sin(x) - 2x + 1 = 0$.

Let $f(x) = \sin(x) - 2x + 1$.

Notice $f(0) = \sin(0) - 2\cdot 0 + 1 = 1 > 0$.

Also, $f(2\pi) = \sin(2\pi) - 2(2\pi) + 1$

$= -4\pi + 1 \approx -12 < 0$.

Since $f(x)$ is continuous, the IVT implies there is an x in $[0, 2\pi]$ with $f(x) = 0$.