

Name: \_\_\_\_\_

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There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [15 points] Compute the derivatives of the following functions.

a.  $f(r) = \frac{3}{r^5}$        $f(r) = 3r^{-5}$   
 $f'(r) = -15r^{-6} = \frac{-15}{r^6}$

b.  $f(x) = -\frac{1}{\sqrt{x}} + 2 + e^x$

$$f(x) = -x^{-1/2} + 2 + e^x \quad f'(x) = \frac{1}{2}x^{-3/2} + e^x$$

c.  $f(x) = \frac{\sqrt{x}+5}{x^2}$ . Hint: Don't bother with the quotient rule.

$$f(x) = x^{1/2-2} + 5x^{-2} \quad f'(x) = -\frac{3}{2}x^{-5/2} - 10x^{-3}$$

$$= x^{-3/2} + 5x^{-2}$$

d.  $f(x) = x^{1/2}e^x$

$$f'(x) = \frac{1}{2}x^{-3/2}e^x + x^{1/2}e^x$$

e.  $f(x) = \frac{x^2-1}{x^2+1}$

$$f'(x) = \frac{2x(x^2+1) - (x^2-1)2x}{(x^2+1)^2}$$

$$= \frac{2x(x^2+1-x^2+1)}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2}$$

2. [5 points] A population of lynx is declining. The population at time  $t$  is

$$P(t) = \frac{800}{2+t}$$

where  $P$  is the number of lynx and where  $t$  is measured in years.

Compute the rate of change of the lynx population, with units, at time  $t = 3$  years.

$$P'(t) = \frac{0 \cdot (2+t) - 800(-1)}{(2+t)^2} = \frac{-800}{(2+t)^2}$$

$$P'(3) = \frac{-800}{25} = -32 \text{ lynx/year}$$

3. [6 points] A particle is moving along a line, and its position  $x$  as a function of time  $t$  is

$$x(t) = (t^2 - 2)e^t.$$

- a. Compute the velocity of the particle.

$$\begin{aligned} x'(t) &= (2t)e^t + (t^2 - 2)e^t \\ &= (t^2 + 2t - 2)e^t \end{aligned}$$

- b. Compute the acceleration of the particle.

$$\begin{aligned} x''(t) &= (2t + 2)e^t + (t^2 + 2t - 2)e^t \\ &= (t^2 + 4t)e^t \end{aligned}$$

4. [4 points] Find the formula for the tangent line to the curve  $y = x - x^2$  at  $x = 3$ .

$$\text{@ } x=3, y = 3 - 3^2 = -6$$

$$y' = 1 - 2x$$

$$\text{@ } x=3, y' = -5$$

$$y + 6 = -5(x - 3)$$

$$y = 6 - 5(x - 3)$$