Name: ______________________________ ________ / 20

There are 20 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[12 points]** Compute the derivatives of the following functions. Simplify your answers.

a. $f(r) = (1 - r^3)\sec(r)$

$$f'(r) = \frac{d}{dr}\left[(1-r^3)\sec(r)\right] = \left[\frac{d}{dr}(1-r^3)\right]\sec(r) + (1-r^3)\frac{d}{dr}\sec(r)$$

$$= -3r^2\sec(r) + (1-r^3)\sec(r)\tan(r)$$

$$= \boxed{\left[-3r^2 + (1-r^3)\tan(r)\right]\sec(r)}$$

b. $f(x) = \dfrac{\cos(x)}{1 - e^{ax}}$, where a is a constant real number.

$$f'(x) = \frac{\left[\frac{d}{dx}\cos(x)\right](1-e^{ax}) - \cos(x)\frac{d}{dx}(1-e^{ax})}{(1-e^{ax})^2} = \boxed{\frac{-\sin(x)(1-e^{ax}) + \cos(x)ae^{ax}}{(1-e^{ax})^2}}$$

c. $f(t) = \sqrt{1 + t^2 e^t}$.

$$f'(t) = \frac{1}{2\sqrt{1+t^2e^t}} \cdot \frac{d}{dt}(1+t^2e^t)$$

$$= \frac{2te^t + t^2e^t}{2\sqrt{1+t^2e^t}} = \boxed{\frac{e^t\left[2t+t^2\right]}{2\sqrt{1+t^2e^t}}}$$

d. $f(x) = \tan\left(x^2 - e^{\sin(x)}\right)$

$$f'(x) = \sec^2\left(x^2 - e^{\sin(x)}\right) \cdot \frac{d}{dx}\left[x^2 - e^{\sin(x)}\right]$$

$$= \boxed{\sec^2\left(x^2 - e^{\sin(x)}\right) \cdot \left(2x - e^{\sin(x)}\cos(x)\right)}$$

2. [4 points] The length of a day in a certain city is given by

$$L(t) = 12 + 8\sin\left(2\pi\frac{t-80}{365}\right).$$

where L is measured in hours and t is measured in days, with $t = 0$ representing January 1.

a. Compute $L'(t)$.

$$L'(t) = 8\cos\left(2\pi\frac{t-80}{365}\right)\cdot\frac{2\pi}{365} = \frac{16\pi}{365}\cos\left(2\pi\frac{t-80}{365}\right)$$

b. Suppose you have computed $L'(220) \approx -0.1$. Interpret what this means in precise language that your parents could nevertheless understand. Your answer must include units for full credit.

On day 220 (sometime in July) the days are getting shorter by 0.1 hours/day = 6 minutes/day

3. [4 points] Determine all times t such that the graph of $y = \sin(3x) - 3x$ has a horizontal tangent.

$\frac{dy}{dx} = 3\cos(3x) - 3$; want $\frac{dy}{dx} = 0$

$3\cos(3x) = 3 \Rightarrow \cos(3x) = 1$

So $3x = 2\pi k$

$x = \frac{2}{3}\pi k$, k an integer.