Name: Solutions ______ / 20

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [5 points]

$f(x) = x^{-1}$

a. Compute the linear approximation of $f(x) = 1/x$ at $x = 10$.

$f(10) = 1/10$

$f'(x) = -x^{-2}$

$f'(10) = \frac{-1}{100}$

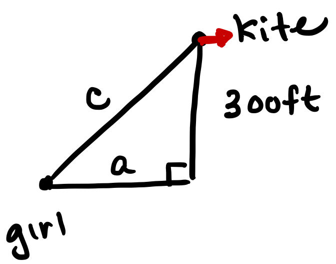
$y - \frac{1}{10} = \frac{-1}{100}(x-10)$

$$\boxed{L(x) = \frac{1}{10} - \frac{1}{100}(x-10)}$$

b. Use your answer above to find a decimal approximation for $1/11$.

$$\frac{1}{11} = f(11) \approx L(11) = \frac{1}{10} - \frac{1}{100}(11-10) = \frac{1}{10} - \frac{1}{100} = 0.1 - 0.01 = \boxed{0.09}$$

2. [8 points] A girl flies a kite at a height of 300 ft. A wind blows the kite horizontally at a rate of 25 ft/sec. How fast must she let out the string for the kite when the kite is 500 ft away from her?



$\frac{da}{dt} = 25 \text{ ft/s}$

Find $\frac{dc}{dt}$ when $c = 500$ft.

$a^2 + 300^2 = c^2$

$2a\frac{da}{dt} = 2c\frac{dc}{dt}$

(need a: when $c = 500$ $a^2 + 300^2 = 500^2$ implies $a = 400$.)

Plug in: $2 \cdot 400 \cdot 25 = 2 \cdot 500 \cdot \frac{dc}{dt}$

$$\frac{dc}{dt} = \frac{10{,}000}{500} = \boxed{20 \text{ ft/s}}$$

← How fast the string must be let out.

3. [8 points] A population of bacteria is growing exponentially. At time $t = 0$ minutes there are 500 bacteria. At time $t = 30$ minutes there are 1200 bacteria. Find an expression for $P(t)$, the population of the bacteria at any time t. Your expression must be such that if you know the time t and you have a calculator, then you can compute the number $P(t)$.

exponential growth means $P = Ce^{kt}$.

two points: $(0, 500)$, $(30, 1200)$.

(plug in 1st pt.)

$500 = Ce^0 = C$.

So $P = 500e^{kt}$

(plug in 2nd pt.)

$1200 = 500e^{k \cdot 30}$

$\frac{12}{5} = e^{k \cdot 30}$

$\ln \frac{12}{5} = 30k$

$\to$ So $k = \frac{\ln(12/5)}{30}$.

Answer: $\boxed{P(t) = 500e^{\left(\frac{\ln(12/5)}{30}\right)t}}$

4. [4 points] The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the base of the cone and h is the height of the cone. Use a differential to estimate the change in volume of the cone if the height is fixed at 9 feet and the radius changes from 5 feet to 5.5 feet.

$V = \frac{1}{3}\pi r^2 h$

$h = 9$. So $\underline{V} = \frac{1}{3}\pi r^2 \cdot 9 = \underline{3\pi r^2}$.

differential:

$dV = 6\pi r\, dr$; given: $dr = 0.5$

$\Delta V \approx dV = 6\pi(5)\frac{1}{2} = \boxed{15\pi \text{ ft}^3}$