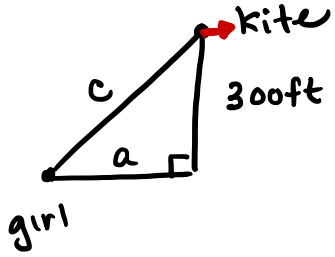
**Name:** _____ _____ / 20

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

**1. [8 points]** A girl flies a kite at a height of 300 ft. A wind blows the kite horizontally at a rate of 20 ft/sec. How fast must she let out the string for the kite when the kite is 500 ft away from her?




$\frac{da}{dt} = 20$ ft/s  Find $\frac{dc}{dt}$ when $c = 500$ ft.

$$a^2 + 300^2 = c^2$$

$$2a\frac{da}{dt} = 2c\frac{dc}{dt}$$

(need a: When $c=500$ $a^2+300^2=500^2$ implies $a=400$.)

Plug in: $2 \cdot 400 \cdot 20 = 2 \cdot 500 \cdot \frac{dc}{dt}$

$$\frac{dc}{dt} = \frac{8000}{500} = \boxed{\frac{80}{5}\text{ ft/s}}$$ ← How fast the string must be let out.

**2. [5 points]**

$f(x) = x^{-1}$

**a.** Compute the linear approximation of $f(x) = 1/x$ at $x = 10$.

$f(10) = 1/10$

$f'(x) = -x^{-2}$

$f'(10) = \frac{-1}{100}$

$y - \frac{1}{10} = \frac{-1}{100}(x-10)$

$$\boxed{L(x) = \frac{1}{10} - \frac{1}{100}(x-10)}$$

**b.** Use your answer above to find a decimal approximation for $1/9$.

$$\frac{1}{9} = f(9) \approx L(9) = \frac{1}{10} - \frac{1}{100}(9-10) = \frac{1}{10} + \frac{1}{100} = 0.1 + 0.01 = \boxed{0.11}$$

**3. [8 points]** A population of bacteria is growing exponentially. At time $t = 0$ minutes there are 400 bacteria. At time $t = 30$ minutes there are 900 bacteria. Find an expression for $P(t)$, the population of the bacteria at any time $t$. Your expression must be such that if you know the time $t$ and you have a calculator, then you can compute the number $P(t)$.

exponential growth means $P = Ce^{kt}$.

two points: $(0, 400)$, $(30, 900)$.

(plug in 1st pt.)

$400 = Ce^{0} = C$.

So $P = 400e^{kt}$

(plug in 2nd pt.)

$900 = 400e^{k \cdot 30}$

$\frac{9}{4} = e^{k \cdot 30}$

$\ln \frac{9}{4} = 30k$

So $k = \frac{\ln(9/4)}{30}$.

Answer: $P(t) = 400e^{\left(\frac{\ln(9/4)}{30}\right)t}$

**4. [4 points]** The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ where $r$ is the radius of the base of the cone and $h$ is the height of the cone. Use a differential to estimate the change in volume of the cone if the height is fixed at 6 feet and the radius changes from 5 feet to 5.5 feet.

$V = \frac{1}{3}\pi r^2 h$

$h = 6$. So $V = \frac{1}{3}\pi r^2 \cdot 6 = 2\pi r^2$.

differential:

$dV = 4\pi r\, dr$; given: $dr = 0.5$

$\Delta V \approx dV = 4\pi(5)\frac{1}{2} = 10\pi \text{ ft}^3$