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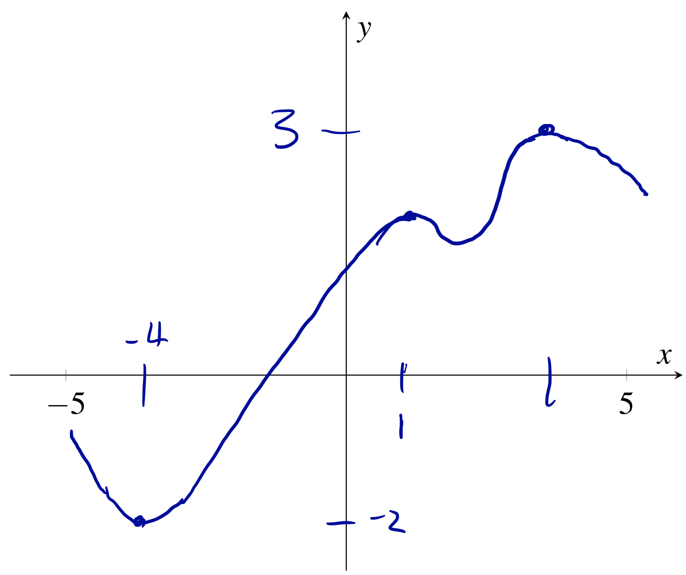
There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [5 points] Sketch a function on $[-5,5]$ that has an absolute maximum value of 3 at $x=4$, an absolute minimum value of -2 at $x=-4$, and a local maximum at $x=1$. You should appropriately label notable values on the x- and y-axes for full credit.



2. [5 points] Find all critical points of the function $f(x) = x(x-1)^{1/3}$. Be careful!

$$f'(x) = (x-1)^{1/3} + \frac{x}{3}(x-1)^{-2/3}$$

$f'(x)$ does not exist at $x=1$

$$f'(x)=0: \quad (x-1)^{1/3} + \frac{x}{3}(x-1)^{-2/3} = 0$$

$$(x-1) + \frac{x}{3} = 0$$

$$\frac{4}{3}x - 1 = 0$$

$$x = \frac{3}{4}$$

$$\boxed{\text{Critical points: } x = 1, \frac{3}{4}}$$

3. [10 points] Find the maximum and minimum values of the function $f(x) = 1/x - 1/x^2$ on the interval $[1, 10]$.

$$f'(x) = \frac{-1}{x^2} + \frac{2}{x^3}$$

$$f'(x) = 0: \quad -\frac{1}{x^2} + \frac{2}{x^3} = 0 \Rightarrow -x + 2 = 0 \Rightarrow x = 2$$

$$f(1) = 1 - 1 = 0$$
$$f(2) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$
$$f(10) = \frac{1}{10} - \frac{1}{100}$$

$\rightarrow$ maximum value: $\frac{1}{4}$ at $x = 2$

minimum value: 0 at $x = 1$

4. [5 points] Suppose f is continuous on $[-2, 2]$ and has a derivative at each point in $(-2, 2)$. Suppose $f(-2) = 6$ and $f(2) = -4$. What does the Mean Value Theorem let you conclude?

There is a point c in $(-2,2)$ where

$$f'(c) = \frac{-4 - 6}{2 - (-2)} = \frac{-10}{4} = -\frac{5}{2}$$

5. [5 points] Draw a diagram that illustrates the Mean Value Theorem in the context of the previous problem. Your illustration should include a tangent line somewhere.