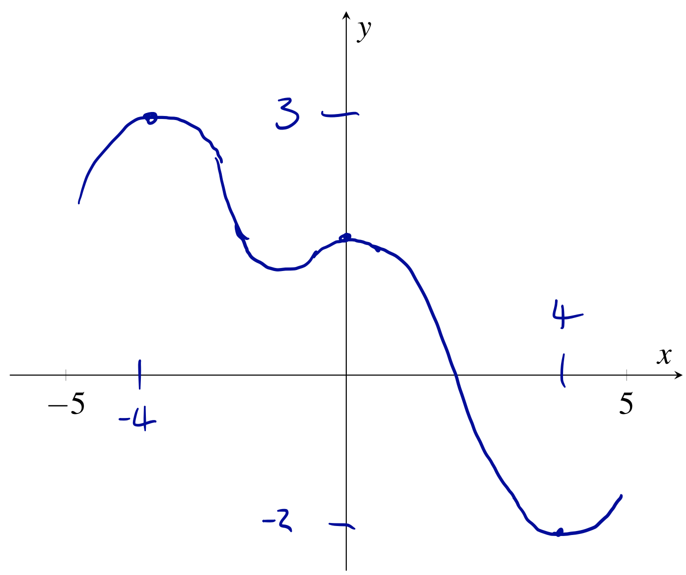
Name: ______________________________ ________ / 30

There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[5 points]** Sketch a function on $[-5, 5]$ that has an absolute maximum value of 3 at $x = -4$, an absolute minimum value of -2 at $x = 4$, and a local minimum at $x = 0$. You should appropriately label notable values on the x- and y-axes for full credit.



2. **[5 points]** Find all critical points of the function $f(x) = x(x-1)^{2/3}$. Be careful!

$$f'(x) = (x-1)^{2/3} + \frac{2x}{3}(x-1)^{-1/3}$$

$f'(x)$ does not exist at $x=1$

$$f'(x) = 0: \quad (x-1)^{2/3} + \frac{2x}{3}(x-1)^{-1/3} = 0$$

$$(x-1) + \frac{2x}{3} = 0$$

$$\frac{5}{3}x - 1 = 0$$

$$x = \frac{3}{5}$$

$$\boxed{\text{Critical points: } x = 1, \frac{3}{5}}$$

3. [10 points] Find the maximum and minimum values of the function $f(x) = 1/x - 2/x^2$ on the interval $[1, 10]$.

$$f'(x) = -\frac{1}{x^2} + \frac{4}{x^3}$$

$$f'(x) = 0: \quad -\frac{1}{x^2} + \frac{4}{x^3} = 0 \Rightarrow -x + 4 = 0$$
$$\Rightarrow x = 4$$

$$f(1) = 1 - 2 = -1$$
$$f(4) = \frac{1}{4} - \frac{2}{16} = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$
$$f(10) = \frac{1}{10} - \frac{1}{100}$$

minimum of -1 at $x = 1$

maximum of $\frac{1}{8}$ at $x = 4$

4. [5 points] Suppose f is continuous on $[-2, 2]$ and has a derivative at each point in $(-2, 2)$. Suppose $f(-2) = -2$ and $f(2) = 3$. What does the Mean Value Theorem let you conclude?

There is a point c in $(-2, 2)$ with $f'(c) = \dfrac{3-(-2)}{2-(-2)} = \dfrac{5}{4}$

5. [5 points] Draw a diagram that illustrates the Mean Value Theorem in the context of the previous problem. Your illustration should include a tangent line somewhere.