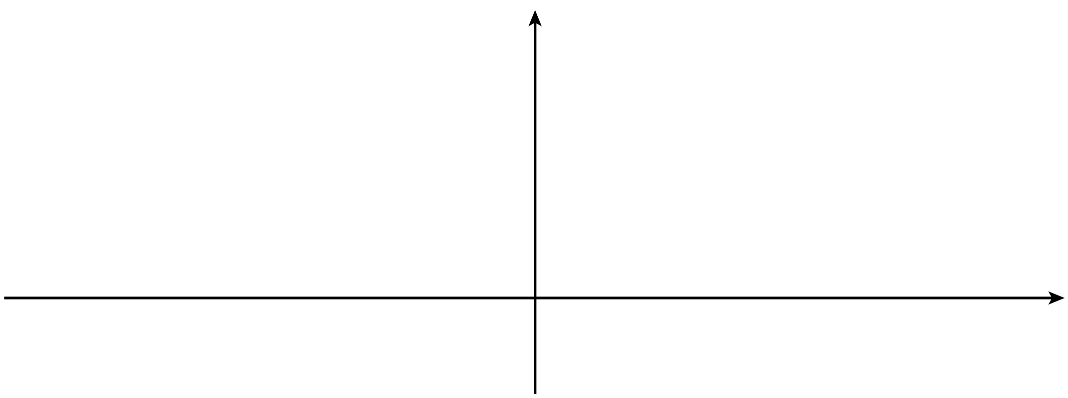
Name: ____ __________ **/ 30**

There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] Sketch the graph of a continuous function with domain $\mathbb{R}$ that satisfies all of the following features.

1. $f(3) = 0$,
2. $f'(x) > 0$ for $x < 0$; $\quad f'(x) < 0$ for x in $(0,3)$; $\quad f'(x) > 0$ for $x > 0$,
3. $f'(0) = f'(3) = 0$,
4. $f''(x) < 0$ for $-1 < x < 1$; $f''(x) > 0$ for $x < -1$ or $x > 1$
5. $\lim_{x\to-\infty} f(x) = 0$; $\quad \lim_{x\to\infty} f(x) = \infty$

Your sketch should label all interesting points on the x-axis. Additionally, place a **small triangle** on the graph at any points of inflection.



2. [6 points] Compute the following limits.

a. $\displaystyle\lim_{x\to1} \frac{x^a - 1}{x^{2b} - 1}$ where a and b are constants, $b \neq 0$.

b. $\displaystyle\lim_{x\to\infty} x^2 e^{-3x}$.

3. [6 points] Consider the function $f(x) = \frac{1}{x} + \ln x$. We have computed for you

$$f'(x) = \frac{x-1}{x^2}; \qquad f''(x) = \frac{2-x}{x^3}.$$

a. Find the intervals where $f(x)$ is increasing and decreasing. [Be careful about the domain of $f(x)$!]

b. Find the intervals where $f(x)$ is concave up and concave down.

4. [8 points] Consider the function $f(x) = x \ln x$.

a. This function has a single critical point c. Find it.

b. Use the First Derivative Test to classify the critical point c from part **a**) as a local minimum/maximum/neither.

c. Use the Second Derivative Test to classify the critical point c from part **a**) as a local minimum or maximum if this is possible (or state that the Second Derivative Test is inconclusive).