**Name:** ———————————————————————————   ——————— / 30

There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[10 points]** Sketch the graph of a continuous function with domain $\mathbb{R}$ that satisfies all of the following features.

   1. $f(-3) = 0$,

   2. $f'(x) < 0$ for $x < -3$; $f'(x) > 0$ for x in $(-3,0)$; $f'(x) < 0$ for $x > 0$,

   3. $f'(-3) = f'(0) = 0$,

   4. $f''(x) < 0$ for $-1 < x < 1$; $f''(x) > 0$ for $x < -1$ or $x > 1$

   5. $\lim\limits_{x \to -\infty} f(x) = \infty$; $\lim\limits_{x \to \infty} f(x) = 0$

   Your sketch should label all interesting points on the x-axis. Additionally, place a **small triangle** on the graph at any points of inflection.
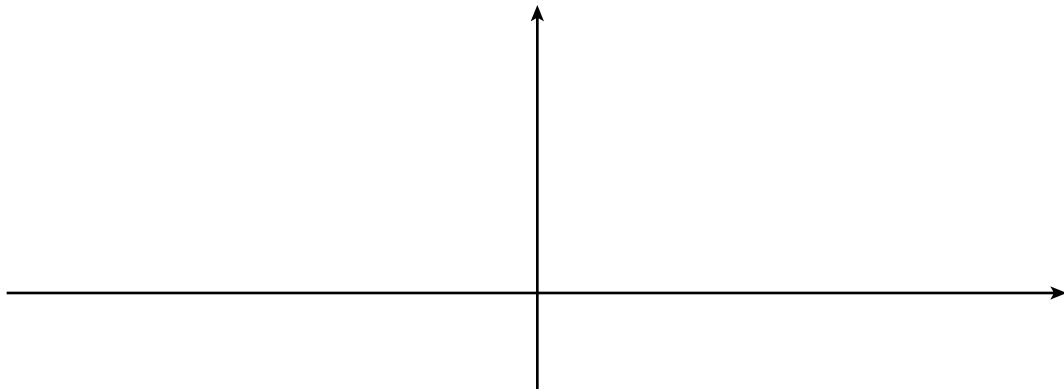
2. **[6 points]** Compute the following limits.

   a. $\lim\limits_{x \to 1} \dfrac{x^{2a} - 1}{x^b - 1}$ where a and b are constants, $b \neq 0$.

   b. $\lim\limits_{x \to \infty} x^2 e^{-4x}$.

3. [6 points] Consider the function $f(x) = \dfrac{2}{x} + \ln x$. We have computed for you

$$f'(x) = \frac{x-2}{x^2}; \qquad f''(x) = \frac{4-x}{x^3}.$$

a. Find the intervals where $f(x)$ is increasing and decreasing. [Be careful about the domain of $f(x)$!]

b. Find the intervals where $f(x)$ is concave up and concave down.

4. [8 points] Consider the function $f(x) = x \ln x$.

a. This function has a single critical point c. Find it.

b. Use the First Derivative Test to classify the critical point c from part **a**) as a local minimum/maximum/neither.

c. Use the Second Derivative Test to classify the critical point c from part **a**) as a local minimum or maximum if this is possible (or state that the Second Derivative Test is inconclusive).