Name: Solutions _____ / **40**

There are 40 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] In each case below, find a function f that satisfies the given criteria.

a. $f'(t) = \cos(t) - 1/t^2$

$f(t) = \sin(t) + \frac{1}{t}$

b. $f''(t) = 5 + 3e^t$, $f(0) = 1$, $f'(0) = -2$

$f'(t) = 5t + 3e^t + C$

$f'(0) = 3 + C = -2$

$\Rightarrow C = -5$

$f(t) = \frac{5}{2}t^2 + 3e^t - 5t + C$

$f(0) = 3e^0 + C = 1$

$\Rightarrow C = -2$

$f(t) = \frac{5}{2}t^2 + 3e^t - 5t - 2$

2. [10 points] Gravel is being added to a pile at a rate of rate of $1 + t^2$ tons per minute for $0 \le t \le 10$ minutes. That is, if $G(t)$ is the amount of gravel (in tons) in the pile at time t, then

$$G'(t) = 1 + t^2.$$

At time $t = 0$ the pile contains 2 tons of gravel.

a. Find an expression for $G(t)$.

$G(t) = t + \frac{1}{3}t^3 + C$

$G(0) = 2 \Rightarrow C = 2$

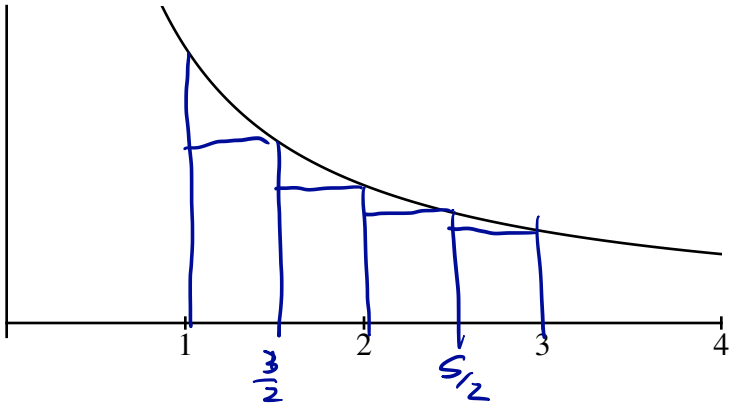
$G(t) = t + \frac{1}{3}t^3 + 2$

b. How much gravel is in the pile at time $t = 10$ minutes?

$G(10) = 10 + \frac{1000}{3} + 2$

$= \frac{1036}{3}$ tons

3. [10 points] Consider the graph of $f(x) = 2/x$ below.



a. Estimate the area under the graph between $x = 1$ and $x = 3$ using four rectangles and right-hand endpoints. Express your answer as a single fraction.

$A \approx \frac{1}{2} \cdot \left(2 \cdot \frac{2}{3} + 2 \cdot \frac{1}{2} + 2 \cdot \frac{2}{5} + 2 \cdot \frac{1}{3}\right)$

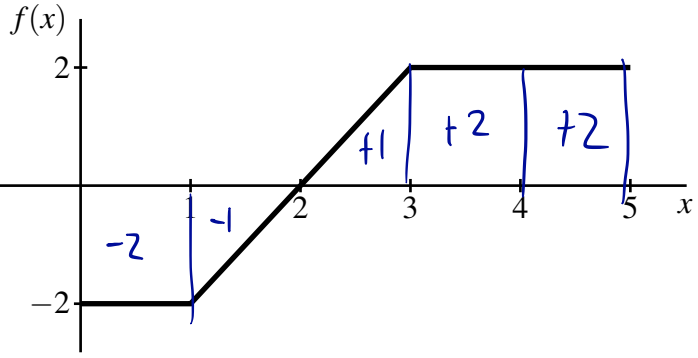
$= \left(\frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3}\right) = \frac{3}{2} + \frac{2}{5} = \frac{19}{10}$

b. In the diagram above, add rectangles to show the area that you actually computed.

c. Is your estimate an overestimate or and underestimate? Briefly justify your answer.

Underestimate. All rectangles are inside the area we are estimating.

4. [10 points] The graph of the function $f(x)$ is shown below.



Evaluate the following integrals using the area interpretation of the integral.

a. $\int_0^2 f(x)\,dx$

$-2 - 1 = -3$

b. $\int_1^3 f(x)\,dx$

0

c. $\int_0^5 f(x)\,dx$

2