Name: Solutions ______ / **40**

There are 40 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] In each case below, find a function f that satisfies the given criteria.

a. $f'(t) = \cos(t) - 1/t^3$

$f(t) = \sin(t) + \frac{1}{2} t^{-2}$

b. $f''(t) = 6 - 2e^t$, $f(0) = 1$, $f'(0) = -3$

$f'(t) = 6t - 2e^t + C$

$f'(0) = -3 \Rightarrow C = -1$

$f(t) = 3t^2 - 2e^t - t + C$

$f(0) = 1 \Rightarrow C = 3$

$f(t) = 3t^2 - 2e^t - t + C$

2. [10 points] Gravel is being added to a pile at a rate of rate of $1+t^3$ tons per minute for $0 \le t \le 10$ minutes. That is, if $G(t)$ is the amount of gravel (in tons) in the pile at time t, then

$$G'(t) = 1 + t^3.$$

At time $t = 0$ the pile contains 3 tons of gravel.

a. Find an expression for $G(t)$.

$G(t) = t + \frac{t^4}{4} + C$

$G(0) = 3 \Rightarrow C = 3$

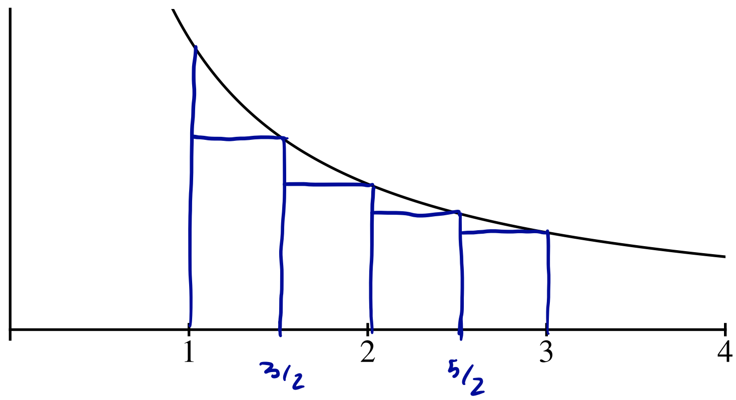
$\boxed{G(t) = t + \frac{t^4}{4} + 3}$

b. How much gravel is in the pile at time $t = 10$ minutes?

$G(10) = 13 + \frac{10000}{4}$

$= 2513$

3. [10 points] Consider the graph of $f(x) = 3/x$ below.



a. Estimate the area under the graph between $x = 1$ and $x = 3$ using four rectangles and right-hand endpoints. Express your answer as a single fraction.

$$A \approx \frac{1}{2}\left(3\cdot\frac{2}{3} + 3\cdot\frac{1}{2} + 3\cdot\frac{2}{5} + 3\cdot\frac{1}{3}\right)$$

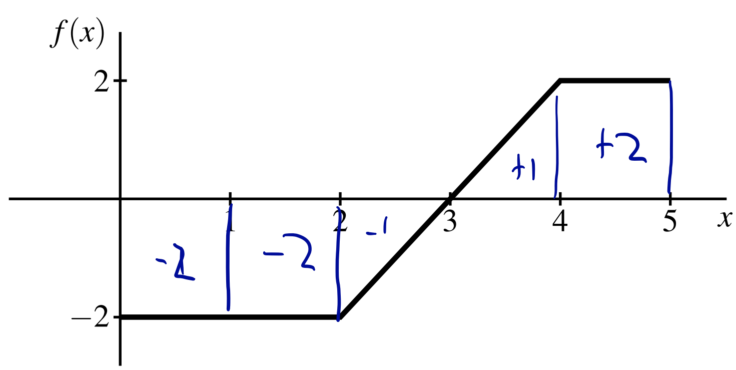
$$= \frac{3}{2}\left(\frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3}\right) = \frac{3}{2}\left(\frac{3}{2} + \frac{2}{5}\right) = \frac{3}{2}\,\frac{19}{10} = \frac{57}{20}$$

b. In the diagram above, add rectangles to show the area that you actually computed.

c. Is your estimate an overestimate or and underestimate? Briefly justify your answer.

Underestimate: the rectangles lie completely within the region we are approximating the area of.

4. [10 points] The graph of the function $f(x)$ is shown below.



Evaluate the following integrals using the area interpretation of the integral.

a. $\int_0^3 f(x)\,dx$

-5

b. $\int_2^4 f(x)\,dx$

0

c. $\int_0^5 f(x)\,dx$

-2