

Name: _____

SOLUTIONS

_____ / 25

Circle one: **Rhodes (F01) | Bueler (F02) | Jurkowski (F03)**

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [4 points] In successive weeks, the amount of heating oil in a tank is recorded, as shown in the table.

| | | | | | | |
|---------------|-----|-----|-----|-----|----|----|
| t (weeks) | 1 | 2 | 3 | 4 | 5 | 6 |
| A (gallons) | 237 | 203 | 157 | 132 | 99 | 62 |

- a. Find the average rate at which the amount changed over the entire period. Specify units.

$$M_{av} = \frac{62 - 237}{6 - 1} = -\frac{175}{5} = -35 \frac{\text{gal}}{\text{week}}$$

$$\begin{array}{r} 237 \\ -62 \\ \hline 175 \end{array}$$

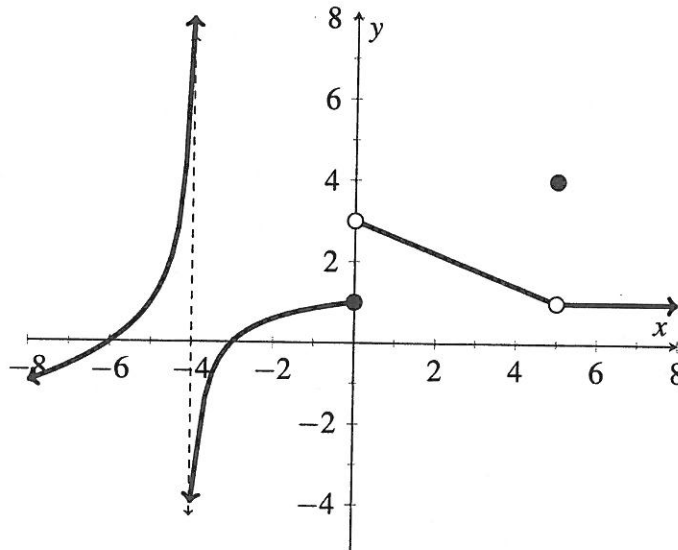
$$\begin{array}{r} 3 \\ 5 \overline{)175} \\ \underline{15} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

- b. Find the average rate of change from week 2 to week 4.

$$M_{av} = \frac{132 - 203}{4 - 2} = -\frac{71}{2} \frac{\text{gal}}{\text{week}}$$

$$\begin{array}{r} 203 \\ 132 \\ \hline 71 \end{array}$$

2. [9 points] Use the graph of the function of $f(x)$ to answer the following questions.



a. $f(-6) = 0$

b. $f(0) = 1$

c. $f(5) = 4$

d. $\lim_{x \rightarrow 0^+} f(x) = 3$

e. $\lim_{x \rightarrow 0^-} f(x) = 1$

f. $\lim_{x \rightarrow 0} f(x) = \text{d.n.e.}$

g. $\lim_{x \rightarrow -4^+} f(x) = -\infty$

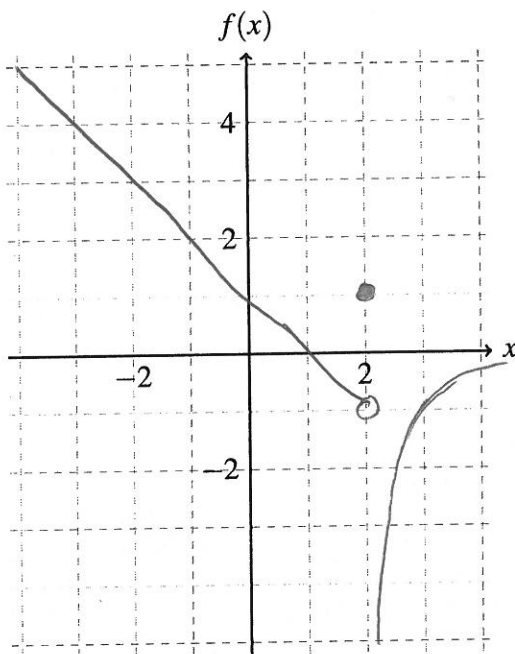
h. $\lim_{x \rightarrow 5} f(x) = 1$

i. $\lim_{x \rightarrow -6} f(x) = 0$

3. [6 points] On the axes below, sketch the graph of the function

$$f(x) = \begin{cases} 1-x & x < 2 \\ 1 & x = 2 \\ \frac{1}{2-x} & x > 2. \end{cases}$$

Then compute, with brief justification, the requested values in the table.



| Value | Justification |
|--|---|
| $f(2) = 1$ | as given above |
| $\lim_{x \rightarrow 2^-} f(x) = -1$ | since $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 1-x = 1-2 = -1$ |
| $\lim_{x \rightarrow 2^+} f(x) = \text{DNE}$ | since $\lim_{x \rightarrow 2^+} f(x) = -\infty$ and these are not the same |

4. [6 points] Compute the following limits. For each limit, justify your answer with a sentence or two.

a. $\lim_{x \rightarrow 2^+} \frac{7+x}{(x-2)^2} = \boxed{\infty}$

As $x \rightarrow 2$ from the right, $7+x \rightarrow 9$ and $(x-2)^2 \rightarrow 0$ but is positive. A number near 9 divided by a smaller and smaller positive number gives larger and larger numbers.

b. $\lim_{x \rightarrow \pi^+} \frac{\sqrt{2}}{\sin(x)} = \boxed{-\infty}$

As $x \rightarrow \pi$ from the right, $\sin(x) \rightarrow 0$ but is negative. $\sqrt{2}$ divided by a negative number approaching 0 will be negative but get larger and larger.