Name: SOLUTIONS ____________ / 25

Circle one: **Rhodes (F01)** | **Bueler (F02)**

25 points possible. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [5 points] Evaluate the limit. Show work and use proper limit notation for full credit.

$$\lim_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h} = \lim_{h\to 0} \frac{\frac{3-(3+h)}{(3+h)3}}{h} = \lim_{h\to 0} \frac{\cancel{3}-\cancel{3}-\cancel{h}}{(3+h)\cdot 3\cdot \cancel{h}}$$

$$= \lim_{h\to 0} \frac{-1}{(3+h)\cdot 3} = \frac{-1}{3\cdot 3} = -\frac{1}{9}$$

2. [5 points] Evaluate the limit. Show work and use proper limit notation for full credit.

$$\lim_{x\to -2} \frac{3x+6}{x^2-4} = \lim_{x\to -2} \frac{3\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \lim_{x\to -2} \frac{3}{x-2}$$

$$= \frac{3}{(-2)-2} = -\frac{3}{4}$$

3. [4 points]

a. Why is the following not a true statement?:

$$\frac{x^2+5x}{x} = x+5$$

The function on the left has a <u>different domain</u> from the function on the right. (Domains are $(-\infty,0)\cup(0,\infty)$ on left and $(-\infty,\infty)$ on right.)

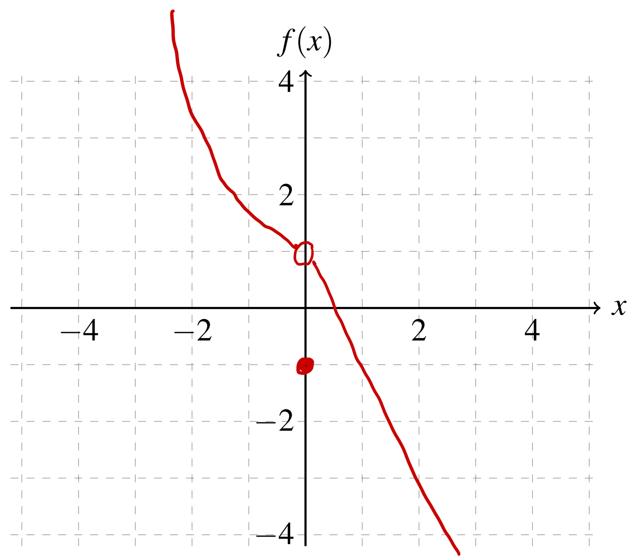
b. Explain why the following equation *is* correct:

$$\lim_{x\to 0} \frac{x^2+5x}{x} = \lim_{x\to 0} x+5$$

Limits "$x\to a$" don't care about the value <u>at</u> a, but only about the function values near a. The two functions <u>agree</u> for values <u>other than $x=0$</u>.

4. [6 points] Consider the function $f(x) = \begin{cases} x^2+1 & \text{if } x<0 \\ -1 & \text{if } x=0 \\ 1-2x & \text{if } x>0 \end{cases}$.

a. On the axes below, sketch a graph of $f(x)$.



b. Evaluate the limit, or explain why it does not exist:

$\lim_{x\to 0} f(x)$ $= 1$ $\left(\text{because } \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2+1 = 1 \text{ and } \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1-2x = 1\right)$

c. Is f continuous at $x=0$? Explain using the definition of continuity.

No. Here $-1 = f(0) \neq \lim_{x\to 0} f(x) = 1$.

5. [5 points] Use the Intermediate Value Theorem to show that there is a root of the equation $x - 3\cos(x) - 6 = 0$ in the interval $(0, \pi)$.

Let $f(x) = x - 3\cos(x) - 6$. Note $f(x)$ is continuous. Also $f(0) = 0-3-6 = -9 < 0$ and $f(\pi) = \pi - 3\cos(\pi) - 6 = \pi + 3 - 6 > 3+3-6 = 0$. By the IVT there is c in $(0,\pi)$ so that $f(c) = 0$.