Name: SOLUTIONS ________ / 25

Circle one: **Rhodes (F01)** | **Bueler (F02)**

25 points possible. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [5 points] Evaluate the limit. Show work and use proper limit notation for full credit.

$$\lim_{x\to-3} \frac{2x+6}{x^2+7x+12} = \lim_{x\to-3} \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x+4)} = \lim_{x\to-3} \frac{2}{x+4}$$

$$= \frac{2}{-3+4} = 2$$

2. [5 points] Evaluate the limit. Show work and use proper limit notation for full credit.

$$\lim_{h\to0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to0} \frac{\frac{2-(2+h)}{(2+h)2}}{h} = \lim_{h\to0} \frac{\cancel{2}-\cancel{2}-h}{(2+h)\cdot2\cdot h}$$

$$= \lim_{h\to0} \frac{-\cancel{h}^{\,1}}{(2+h)\cdot2\cdot\cancel{h}} = \lim_{h\to0} \frac{-1}{(2+h)2} = \frac{-1}{2\cdot2} = \frac{-1}{4}$$

3. [4 points]

a. Why is the following not a true statement?:

$$\frac{x^2-7x}{x} = x-7$$

The function on the left has a <u>different domain</u> from the one on the right. (On the left it is $(-\infty,0)\cup(0,\infty)$. On the right it is $(-\infty,\infty)$.)

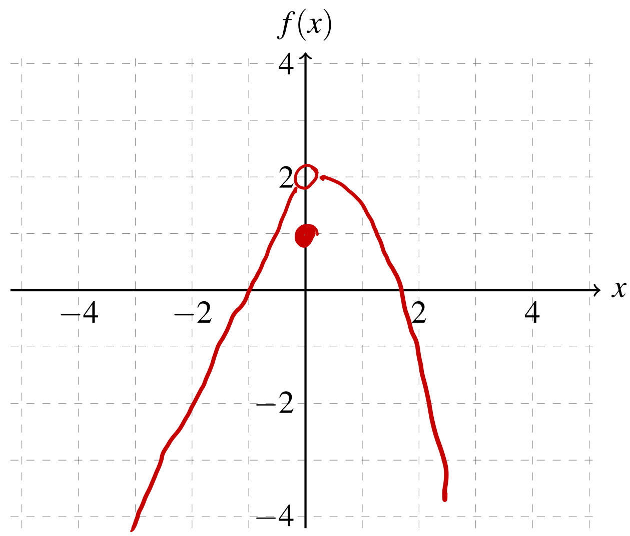
b. Explain why the following equation *is* correct:

$$\lim_{x\to0} \frac{x^2-7x}{x} = \lim_{x\to0} x-7$$

Limits "$x\to a$" don't care about the value <u>at</u> a, but only about the function values near a. The two functions agree <u>for values other than</u> <u>$x=0$</u>.

4. [6 points] Consider the function $f(x) = \begin{cases} 2+2x & \text{if } x<0 \\ 1 & \text{if } x=0 \\ -x^2+2 & \text{if } x>0 \end{cases}$.

a. On the axes below, sketch a graph of $f(x)$.



b. Evaluate the limit, or explain why it does not exist:

$\lim_{x\to 0} f(x) = 2$ $\left(\text{because } \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2+2x = 2 \right.$

$\left. \text{and } \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -x^2+2 = 2 \right)$

c. Is f continuous at $x=0$? Explain using the definition of continuity.

No. Here $1 = f(0) \neq \lim_{x\to 0} f(x) = 2$.

5. [5 points] Use the Intermediate Value Theorem to show that there is a root of the equation $5 - x + 2\cos(x) = 0$ in the interval $(0,\pi)$.

Let $f(x) = 5 - x + 2\cos(x)$. Then $f(x)$ is continuous. Also

$f(0) = 5 - 0 + 2\cos(0) = 5 - 0 + 2 = 7 > 0$ while

$f(\pi) = 5 - \pi + 2(-1) = 5 - 2 - \pi < 5 - 2 - 3 = 0$.

By the IVT there is c in $(0,\pi)$ so that $f(c) = 0$.