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Circle one: **Rhodes (F01)** | **Bueler (F02)**

25 points possible. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[15 points]** Compute the derivatives of the following functions. Write your answer using appropriate derivative notation, but you need not simplify your answers.

   a. $f(x) = 2e^x - x^e + e^2$

   $f'(x) = 2e^x - ex^{e-1}$

   b. $r(x) = \dfrac{3}{x^2} = 3x^{-2}$

   $\dfrac{dr}{dx} = -6x^{-3} = \dfrac{-6}{x^3}$

   c. $g(u) = u^{1/3} - u^{7/3}$

   $g'(u) = \frac{1}{3}u^{-2/3} - \frac{7}{3}u^{4/3}$

   d. $s(t) = (\sqrt{t}+1)\,e^t = (t^{1/2}+1)e^t$

   $\dfrac{ds}{dt} = \frac{1}{2}t^{-1/2}e^t + (t^{1/2}+1)e^t$

   $= \left(\frac{1}{2\sqrt{t}} + \sqrt{t} + 1\right)e^t$

   e. $y = \dfrac{5x^2}{1-2x^3}$

   $y' = \dfrac{10x(1-2x^3) - 5x^2(-6x^2)}{(1-2x^3)^2} = \dfrac{10x - 20x^4 + 30x^4}{(1-2x^3)^2} = \dfrac{10x + 10x^4}{(1-2x^3)^2}$

2. [3 points] Find an equation of the tangent line to the curve $y = 2x - x^2$ at $x = -1$.

$y' = 2 - 2x$

$y'|_{x=-1} = 2 - 2(-1) = 4$

$y|_{x=-1} = 2(-1) - (-1)^2 = -3$

$y - (-3) = 4(x - (-1))$

$y + 3 = 4(x+1)$

$y = 4x + 1$

3. [4 points] Suppose that $f(4) = 2$, $g(4) = 4$, $f'(4) = -1$, and $g'(4) = 3$. Find the following values.

a. $(fg)'(4) = f'(4)g(4) + f(4)g'(4)$

$= (-1)\cdot 4 + 2\cdot 3 = 2$

b. $\left(\frac{f}{g}\right)'(4) = \frac{f'(4)g(4) - f(4)g'(4)}{(g(4))^2} = \frac{(-1)4 - 2\cdot 3}{4^2} = \frac{-10}{16} = \frac{-5}{8}$

4. [3 points] At what x value is the tangent line to the curve $y = e^x - 3x - 2$ parallel to $y = 2x - \frac{3}{2}$?

$\frac{d}{dx}(e^x - 3x - 2) = e^x - 3$

$\frac{d}{dx}(2x - 3/2) = 2$

$e^x - 3 = 2$

$e^x = 5$

$x = \ln 5$