Name: Solutions _______ / 25

Circle one: Rhodes (F01) | Bueler (F02)

25 points possible. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[15 points]** Compute the derivatives of the following functions. Write your answer using appropriate derivative notation, but you need not simplify your answers.

a. $f(x) = 3e^x - x^e + e^3$

$f'(x) = 3e^x - e x^{e-1}$

b. $g(u) = u^{2/3} - u^{5/3}$

$\frac{dg}{du} = \frac{2}{3} u^{-1/3} - \frac{5}{3} u^{2/3}$

c. $r(x) = \frac{2}{x^3} = 2x^{-3}$

$r'(x) = -6x^{-4} = -\frac{6}{x^4}$

d. $s(t) = e^t\left(\sqrt{t} - 1\right) = e^t(t^{1/2} - 1)$

$\frac{ds}{dt} = e^t(t^{1/2} - 1) + e^t\left(\frac{1}{2}t^{-1/2}\right)$

$= e^t\left(\sqrt{t} - 1 + \frac{1}{2\sqrt{t}}\right)$

e. $y = \frac{2x^2}{1 - 5x^3}$

$y' = \frac{4x(1-5x^3) - 2x^2(-15x^2)}{(1-5x^3)^2} = \frac{4x - 20x^4 + 30x^4}{(1-5x^3)^2} = \frac{4x + 10x^4}{(1-5x^3)^2}$

2. **[4 points]** Suppose that $f(3)=2$, $g(3)=4$, $f'(3)=-1$, and $g'(3)=3$. Find the following values.

a. $(fg)'(3) = f'(3)g(3)+f(3)g'(3)$

$= (-1)(4)+(2)(3) = 2$

b. $\left(\frac{f}{g}\right)'(3) = \frac{f'(3)g(3)-f(3)g'(3)}{(g(3))^2} = \frac{-1\cdot 4-2\cdot 3}{4^2} = \frac{-10}{16} = \frac{-5}{8}$

3. **[3 points]** Find an equation of the tangent line to the curve $y=2x-x^2$ at $x=-1$.

$y' = 2-2x$

$y'\big|_{x=-1} = 2-2(-1) = 4$

$y\big|_{x=-1} = 2(-1)-(-1)^2 = -3$

$y-(-3) = 4(x-(-1))$

$y+3 = 4(x+1)$

$y = 4x+1$

4. **[3 points]** At what x value is the tangent line to the curve $y=e^x-2x-3$ parallel to $y=3x-\frac{5}{2}$?

$\frac{d}{dx}(e^x-2x-3) = e^x-2$

$\frac{d}{dx}(3x-5/2) = 3$

$e^x-2=3$

$e^x=5$

$x=\ln 5$

V-2