Math 251: Quiz 6 March 19, 2019

Name: Solutions

Circle one: Rhodes (F01) | Bueler (F02)

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25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [7 points]

a. Give a linear approximation to the function $f(x) = \sqrt{x}$ for x near 25.

$f(25) = 5$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(5) = \frac{1}{2}\frac{1}{\sqrt{25}} = \frac{1}{10}$

$\sqrt{x} \approx 5 + \frac{1}{10}(x-25)$

b. Use your approximation to estimate $\sqrt{24}$.

$\sqrt{24} \approx 5 + \frac{1}{10}(-1) = 4.9$

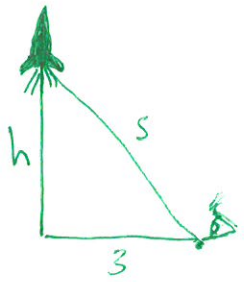
2. [6 points] An invasive plant species is introduced in the middle of a large flat region, and spreads outward over time in a circular pattern, with the radius growing at a rate of 2 *km/year*. How fast is the plant-covered area growing when the radius is 30 *km*? Indicate appropriate units.

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(30)\cdot 2 = 120\pi \frac{km^2}{year}$

3. **[6 points]** A rocket is launched vertically upward, and tracked by a ground observer located 3 km from the launch pad. If the rocket is traveling 350 *km/hour* when it has reached an altitude of 4 *km*, at what rate is its distance to the observer changing at that moment? Indicate appropriate units.



$s^2 = h^2 + 3^2$

$2s\frac{ds}{dt} = 2h\frac{dh}{dt}$ $\qquad \frac{dh}{dt} = 350\ \frac{km}{hr}$

When $h=4$, $s^2 = 16+9 = 25$, so $s = 5$

$2(5)\frac{ds}{dt} = 2(4)350$

$\frac{ds}{dt} = \frac{4}{5}(350) = 280\ km/hr$

4. **[6 points]** A population of 2 thousand cells of algae is introduced into a large vat of growing medium. After 3 days, the population has grown to 30 thousand cells. Assuming the population grows at a rate proportional to the size of the population, give a formula for the size of the population after t days. (Your answer may involve exponentials or logarithms but should have no unspecified constants.)

$P(t) = Ce^{rt}$ population size (in thousands)

$P(0) = 2 \Rightarrow 2 = Ce^{r\cdot 0} \Rightarrow C = 2$

$P(3) = 30 \Rightarrow 30 = 2e^{r\cdot 3}$

$15 = e^{3r}$

$\ln(15) = 3r$

$r = \frac{\ln(15)}{3}$

$P(t) = 2e^{\frac{\ln(15)}{3}t}$