Math 251: Quiz 6 Solutions **March 19, 2019**

Name: _____ **_____ / 25**

Circle one: Rhodes (F01) | Bueler (F02)

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. **[6 points]** An invasive plant species is introduced in the middle of a large flat region, and spreads outward over time in a circular pattern with the radius growing at a rate of 3 *km/year*. How fast is the plant-covered area growing when the radius is 40 *km*? Indicate appropriate units.

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(40)3 = 240\pi \ \frac{km^2}{yr}$$

2. **[7 points]**

   a. Give a linear approximation to the function $f(x) = \sqrt{x}$ for x near 16.

$$f(16) = 4$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

$$f'(16) = \frac{1}{2}\frac{1}{\sqrt{16}} = \frac{1}{8}$$

$$\sqrt{x} \approx 4 + \frac{1}{8}(x-16)$$

   b. Use your approximation to estimate $\sqrt{15}$.

$$\sqrt{15} \approx 4 + \frac{1}{8}(-1) = 4 - .125 = 3.875$$

3. **[6 points]** A population of 3 thousand cells of algae is introduced into a large vat of growing medium. After 2 days, the population has grown to 20 thousand cells. Assuming the population grows at a rate proportional to the size of the population, give a formula for the size of the population after t days. (Your answer may involve exponentials or logarithms but should have no unspecified constants.)

$P(t) = Ce^{rt}$ population size (in thousands)

$P(0) = 3 \Rightarrow 3 = Ce^{r\cdot 0} = C$

$P(2) = 20 \Rightarrow 20 = 3e^{r\cdot 2}$

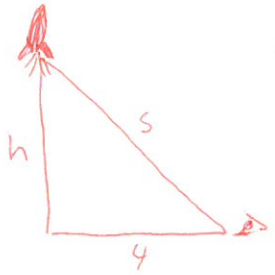
$\frac{20}{3} = e^{2r}$

$\ln\left(\frac{20}{3}\right) = 2r$

$r = \frac{\ln\left(\frac{20}{3}\right)}{2}$

$$P(t) = 3e^{\frac{\ln\left(\frac{20}{3}\right)}{2}t}$$

4. **[6 points]** A rocket is launched vertically upward, and tracked by a ground observer located 4 km from the launch pad. If the rocket is traveling 450 *km/hour* when it has reached an altitude of 3 *km*, at what rate is its distance to the observer changing at that moment? Indicate appropriate units.



$s^2 = h^2 + 4^2$

$2s\frac{ds}{dt} = 2h\frac{dh}{dt}$ $\quad \frac{dh}{dt} = 450\ \frac{km}{hr}$

When $h = 3$, $s^2 = 3^2 + 4^2 = 25$ so $s = 5$

$2(5)\frac{ds}{dt} = 2(3)\,450$

$\frac{ds}{dt} = \frac{3}{5}(450) = 270\ km/hr$