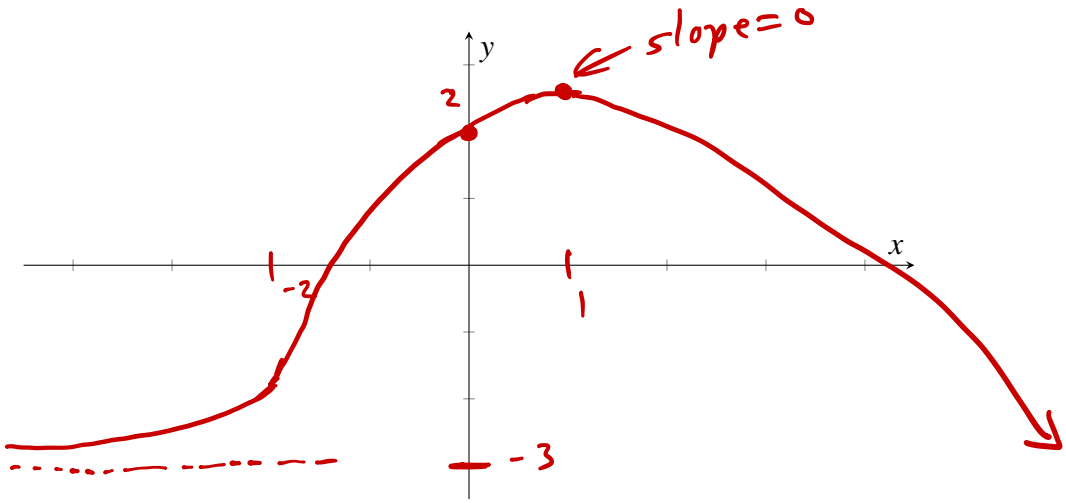
Name: _____ _____ / 25

Circle one: **Rhodes (F01)** | **Bueler (F02)**

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [8 points] Sketch an appropriately labeled graph of a function that satisfies all of the given conditions.

1. $f(0) = 2$
2. $f'(1) = 0$
3. $f'(x) > 0$ for $x < 1$; $f'(x) < 0$ for $x > 1$
4. $f''(x) > 0$ for $x < -2$; $f''(x) < 0$ for $x > -2$
5. $\lim_{x\to-\infty} f(x) = -3$; $\lim_{x\to\infty} f(x) = -\infty$



2. [4 points] Compute the following limits.

a. $\lim_{x\to\infty} \frac{\sqrt{x}}{e^{2x}}$ $\overset{\infty/\infty}{\underset{\text{L'H}}{=}}$ $\lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{2e^{2x}} = \lim_{x\to\infty} \frac{1}{4e^{2x}\sqrt{x}} = 0$

b. $\lim_{x\to 0} \frac{x^2}{e^x - 2} = \frac{0^2}{e^0 - 2} = \frac{0}{1-2} = 0$

3. [13 points] Consider the function $f(x) = \ln(x^2+9)$. We have computed for you

$$f'(x) = \frac{2x}{x^2+9}, \qquad f''(x) = \frac{-2x^2+18}{(x^2+9)^2}.$$

a. Find the domain of $f(x)$.

$x^2+9>0$ ✓ $(-\infty, \infty)$

b. Find intercepts.

$\ln(x^2+9) \geq \ln(9) > 0$ so no x-intercepts

$y = \ln(9)$ ← y-intercept

c. Find the critical point(s).

$f'(x) = 0 = \frac{2x}{x^2+9}$ $\therefore$ $x=0$

d. Determine the intervals where $f(x)$ is increasing and decreasing.

increasing on $[0, \infty)$

decreasing on $(-\infty, 0]$

e. Find the intervals where $f(x)$ is concave up and concave down.

$-2x^2+18=0$

$x^2=9$

$x=\pm 3$

Concave up on $(-3, 3)$

Concave down on $(-\infty, -3) \cup (3, \infty)$

f. Using the above information, sketch the graph of $f(x)$, making sure to label x-coordinates of all important points. [Hint: $\ln 9 \approx 2$]