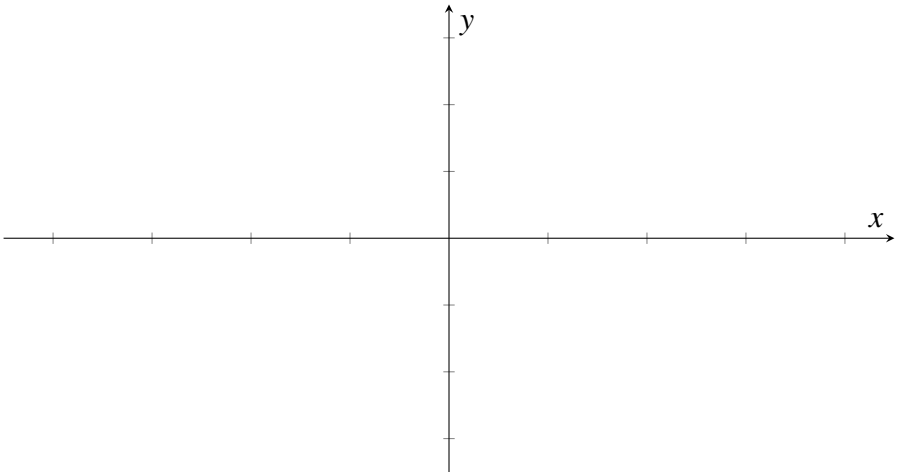
Name: _____ _____ **/ 25**

Circle one: Rhodes (F01) | Bueler (F02)

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [8 points] Sketch an appropriately labeled graph of a function that satisfies all of the given conditions.

1. $f(0) = -2$
2. $f'(1) = 0$
3. $f'(x) < 0$ for $x < 1$; $\quad f'(x) > 0$ for $x > 1$
4. $f''(x) < 0$ for $x < -2$; $\quad f''(x) > 0$ for $x > -2$
5. $\lim_{x\to-\infty} f(x) = 3$; $\quad \lim_{x\to\infty} f(x) = \infty$



2. [4 points] Compute the following limits.

a. $\lim_{x\to\infty} \dfrac{\sqrt{x}}{e^{3x}}$

b. $\lim_{x\to 0} \dfrac{x^2}{2 - e^x}$

3. [13 points] Consider the function $f(x) = \ln(x^2+4)$. We have computed for you

$$f'(x) = \frac{2x}{x^2+4}, \qquad f''(x) = \frac{-2x^2+8}{(x^2+4)^2}.$$

a. Find the domain of $f(x)$.

b. Find intercepts.

c. Find the critical point(s).

d. Determine the intervals where $f(x)$ is increasing and decreasing.

e. Find the intervals where $f(x)$ is concave up and concave down.

f. Using the above information, sketch the graph of $f(x)$, making sure to label x-coordinates of all important points. [Hint: $\ln 4 \approx 1.5$]