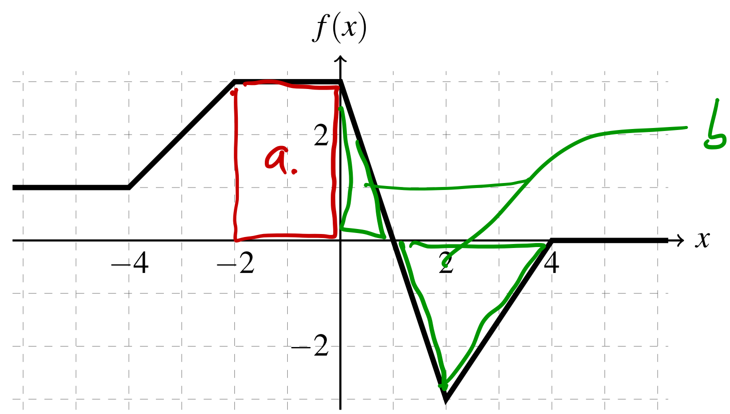
Math 251: Quiz 9 SOLUTIONS **April 16, 2019**

Name: ______________________ ________ **/ 25**

Circle one: **Rhodes (F01)** | **Bueler (F02)**

25 points possible. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit.

1. [6 points] The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



a. $\int_{-2}^{0} f(x)\,dx = 3 \cdot 2 = 6$

b. $\int_{4}^{0} f(x)\,dx = -\int_0^4 f(x)\,dx = -\left(\frac{1}{2}\cdot 3 \cdot 1 - \frac{1}{2}\cdot 3 \cdot 3\right) = 3$

c. $\int_{-2}^{4} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_0^4 f(x)\,dx = 6 - 3 = 3$

2. [6 points] A particle is moving with the given acceleration $a(t)$ and other data. Find the position $s(t)$ of the particle.

$a(t) = 3\cos t - 2\sin t, \quad s(0) = 0, \quad v(0) = 4$

$s''(t) = 3\cos t - 2\sin t$

$s'(t) = 3\sin t + 2\cos t + c$, but: $s'(0) = v(0) = 4$

$3 \cdot 0 + 2 \cdot 1 + c = 4$

$c = 2$

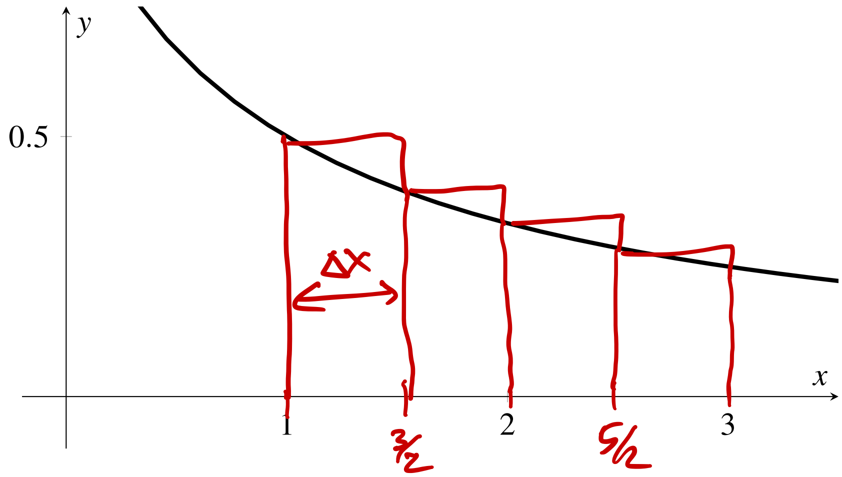
$s(t) = -3\cos t + 2\sin t + 2t + K$, but: $s(0) = 0$

$-3 \cdot 1 + 2 \cdot 0 + 2 \cdot 0 + K = 0$

$K = 3$

$s(t) = -3\cos t + 2\sin t + 2t + 3$

3. [8 points] Consider the graph of $f(x) = \dfrac{1}{1+x}$ below.



$\Delta x = \frac{1}{2}$

a. In the figure above, sketch four rectangles corresponding to the $n = 4$ Riemann sum on the interval $1 \le x \le 3$. Use left endpoints.

$f(1) = \frac{1}{2}$, $f(\frac{3}{2}) = \frac{1}{1+\frac{3}{2}} = \frac{2}{5}$, $f(2) = \frac{1}{3}$, $f(\frac{5}{2}) = \frac{2}{7}$

b. Compute the numerical value of the Riemann sum illustrated in part **a**. Express your answer as a single fraction.

$$L_4 = f(1)\cdot\tfrac{1}{2} + f(\tfrac{3}{2})\cdot\tfrac{1}{2} + f(2)\cdot\tfrac{1}{2} + f(\tfrac{5}{2})\cdot\tfrac{1}{2}$$
$$= \tfrac{1}{2}\left(\tfrac{1}{2} + \tfrac{2}{5} + \tfrac{1}{3} + \tfrac{2}{7}\right) = \tfrac{1}{2}\left(\frac{105+84+70+60}{210}\right)$$
$$= \frac{319}{420}$$

c. Is your numerical value in part **b** an overestimate or an underestimate of $\displaystyle\int_1^3 \frac{1}{1+x}\,dx$?

Over estimate

4. [5 points] Use the Midpoint Rule with $n = 2$ subintervals to approximate the integral. Express your answer as a single fraction.

$$\int_0^4 x2^{-x}\,dx \approx 2\cdot f(1) + 2\cdot f(3)$$
$$= 2\cdot(1\cdot 2^{-1}) + 2\cdot(3\cdot 2^{-3})$$
$$= \frac{2}{2} + \frac{2\cdot 3}{2^3} = \frac{7}{4}$$

$f(x) = x2^{-x}$